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ON THE INTERPRETATION OF PHOTOSPHERIC PHENOMENA

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It is a common belief that a body always presenting the appearance of a circular disk, from whichever side it is looked at, must be bounded by a spherical surface. The general conviction that the bulk of the sun is an incandescent sphere rests on that belief, and was a natural starting-point for solar theories.

After the effective solar temperature had been found so high as to exceed the critical temperatures of perhaps all known substances, the earlier idea that the main body of the sun was in the liquid or the solid state had to be replaced by the hypothesis that it is substantially gaseous. This new idea involved the necessity of explaining the phenomenon of the apparent "solar surface." One had to choose between Young's view, that the photosphere was a layer of incandescent clouds produced by condensation of certain substances having exceptionally high critical temperatures, and Secchi's hypothesis (afterward developed by Schwarzschild and Emden), which dispenses with assuming cloud-formation by supposing the density of the solar gases to increase so rapidly with depth near the level called "solar surface," that within a layer no more than a thousand kilometers thick, their united radiating power increases from a very low value (in the chromosphere) up to that of the black body (in the photosphere).

In 1891 August Schmidt took a new departure when showing that an entirely gaseous body of the dimensions of the sun, in which the density and the radiating power gradually decrease from the center outward, be it even at a slow rate, must appear like a circular luminous disk with a sharp edge, as a mere consequence of ray-curving caused by the radial density gradient. So the circular aspect of the sun is *not* a sufficient ground for admitting the existence of a real "photosphere," that is, of a layer characterized by some abrupt, or even only rapid, change of physical properties.

Schmidt's well-known solar theory, however, met with the severe objection that it did not duly consider the effect of absorption and scattering of the light.[1] Rays having accomplished such long distances on their spiral paths inside the critical sphere would be almost wholly extinguished before emerging; they could not possibly bring along so much energy from the incandescent core as would be required in order to account for the brilliancy observed in the marginal parts of the disk. In its original form the optical interpretation of the sun's edge cannot be maintained.

It is also impossible to accept the cloud-theory of the photosphere, because the results of the radiation-measurements made at Maastricht during the annular eclipse of 1912[2] forbid making an absorbing or scattering solar atmosphere responsible for the fall of the sun's brightness from the center toward the limb. Indeed, the absorbing and scattering power of the gases lying outside the photosphere proved to be relatively insignificant. The photosphere, therefore, cannot be of such a nature that it would appear like a uniformly luminous disk if the surrounding gases were absent. On the contrary, it must have in itself the property of appearing much brighter when looked at in the direction of a radius than at an angle with the radius; and the law of variation of brightness with the angle is different for different wave-lengths.

Whatever the causes may be that make the sun radiate more intensely in the direction of the radius than in directions slanting to it, they must be looked for in layers lying *below* the level generally called the surface of the photosphere. Those layers consist of

[1] R. Emden, *Gaskugeln*, pp. 388–394; Pringsheim, *Physik der Sonne*, pp. 266–270.

[2] *Astrophysical Journal*, **37**, 225, 1913.

transparent gases, for the slightest haze of condensation products, occupying a stratum some thousand kilometers thick, would provide it with a radiating and scattering power almost independent of direction, which power the photosphere does not possess.

Assuming, on the basis of the Maastricht results, that the extinction effected by the sun's outer layers is comparatively small, we derive, from direct observations on the distribution of brightness on the sun's disk (Vogel, Abbott), how much light of a given wave-length a point M, lying somewhere in the photospheric level, transmits on the average along the various directions. The result

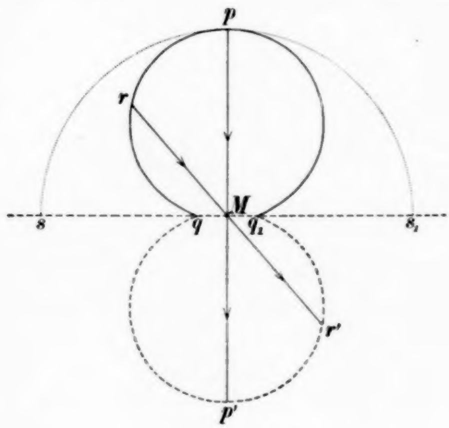

FIG. 1

may conveniently be described, for every wave-length separately, by means of an "irradiation surface" qpq_1 (Fig. 1),[1] the radii vectores of which represent the average intensities of the light reaching M from different sides. We obtain the "radiation or emission surface" $qp'q_1$ of M by prolonging the radii rM and making $Mr'=rM$.

If we wish to explain the sun's apparent, fairly sharp, boundary, and the law of varying brightness of the solar disk, we shall have to consider, besides emission and absorption, the effects of dispersion, refraction, and molecular scattering of the light traversing an

[1] For a method of constructing these surfaces we refer to *Physik. Zeitschr.*, **12**, 677, 1911; or, *Handwörterbuch der Naturwissenschaften*, VII, 830.

entirely gaseous medium. This is a great physical problem, toward the complete solution of which only the first steps are being made as yet;[1] but awaiting the final results of such investigations, we may already inquire into their possible bearing on solar problems.

From the astrophysical point of view one of the questions material to the case is: What can be presumed about the general radial gradient of the density in the layers we are concerned with?

This subject has been treated very fully and ingeniously, on the basis of thermodynamics, by Emden in his book *Gaskugeln*. Emden arrives at the conclusion already mentioned above, that the fall of the density must be extremely rapid; but the inference is open to doubt, for in his calculations Emden presupposes gravitation to be the only radial force acting on solar matter. According to the present state of our physical knowledge, however, we decidedly must admit that on the sun gravitation is counteracted by the pressure of radiation, and by the emission of electrons and perhaps of other charged particles.

Basing on purely theoretical grounds an estimate of the intensity of that counteraction would, for the present, be as rash as denying its existence; but some evidence in favor of its essentiality is given by the fact that many solar phenomena are much better understood if we assume a radial gradient many times smaller than the one that would correspond to gravitational conditions only. In this connection we call attention to the puzzling properties of quiescent, hovering prominences. Father Fényi, in his interesting discussion of the long series of prominence observations made at Haynald Observatory, Kalocsa,[2] is very positive in his assertion that several well-established facts concerning quiet prominences can be accounted for only if in the solar atmosphere gravity is reduced, by certain repulsive forces, to a small fraction (something of the order 1/80) of its commonly accepted value.

[1] Rayleigh, *Phil. Mag.* (5), **47**, 375, 1899; A. Schuster, *Astrophysical Journal*, **21**, 1, 1905; H. A. Lorentz, *The Theory of Electrons*, Leipzig, 1909; L. Natanson, *Bulletin de l'académie des science de Cracovie*, Avril 1907, Décembre 1909; W. H. Julius, *Physik. Zeitschr.*, **12**, 329 and 674, 1911; L. V. King, *Phil. Trans. Roy. Soc. London*, **212** A, 375, 1912.

[2] *Publikationen des Haynald Observatoriums*, Heft X, 138, 1911; cf. also Fényi, "Ueber die Höhe der Sonnenatmosphäre," *Mem. Spettr. ital.* (2), **1**, 21, 1912.

Our hypothesis, that a similar counteraction, opposing the effect of gravitation, prevails throughout the visible layers of the sun, is certainly not less plausible, therefore, than the exclusive hypothesis, usually admitted, which makes gravitation the only effective agent in determining the radial gradient.[1]

We must now endeavor to conceive the appearance of the sun's edge in a transparent gaseous medium where the pressure varies but slowly along the radius.

As already remarked, Schmidt's ingenious optical explanation cannot be adhered to. Nevertheless the principle of ray-curving introduced by that author is extremely suggestive; it leads to the following interpretation of the solar limb, that appears not to encounter similar difficulties.

Let Fig. 2 represent an equatorial section of the sun. It can hardly be doubted that besides the gradual, perhaps slow variation of optical density corresponding to the outward decrease of pressure, there are many *irregular optical density-gradients* connected not only with the local differences of pressure that accompany the convection currents and solar vortices, but also with the differences of temperature and of composition occurring in the gaseous mixture.

Now, the average magnitude of those irregular gradients of optical density will very probably decrease as we proceed from a level P toward a level Q.

Let us imagine the "irradiation surfaces" to be constructed for a point P_1 of the level P and for a point Q_1 of Q. At the level Q the irregular gradients may in general be so small that rays, leaving it along a tangent Q_1E in the direction of the earth, are almost

[1] In this *Journal* (**31**, 166, 1910), Mr. J. A. Anderson has criticized the conclusions arrived at in my paper "Regular Consequences of Irregular Refraction in the Sun" (*Proc. Roy. Acad. Amst.*, October 28, 1909). His refutation of the idea that refraction might be very momentous in solar physics is entirely founded on the following two assumptions: (1) the photosphere may be represented by a perfectly uniform self-luminous surface, radiating approximately according to the cosine law, and (2) on the sun the weight of a gas is 27.3 times as great as on the earth. I think we may now safely state that the first assumption is contrary to observed facts, and that the second assumption is an unproved dogma, subject to well-founded doubts.

Moreover, a very important point, overlooked by Mr. Anderson, is that considerable optical density-gradients may result from differences of temperature or of composition, even at uniform pressure.

never sufficiently curved to be the continuation of rays coming from within the irradiation surface of Q_1. This condition will obtain if the average radius of curvature of rays tangent to the level Q is more than, say, three times as great as the radius of the sphere Q. Then the observer receives little light from Q_1; he will consider the level Q to lie outside the solar limb.

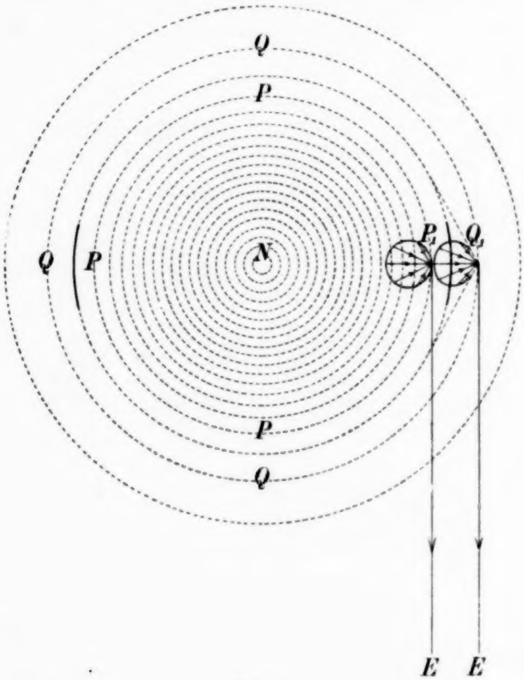

FIG. 2

If, on the other hand, in a layer P the gradients are so much steeper that there the average radius of curvature of tangential rays is smaller than, say, one-third of the radius of the sphere P, we may expect a sensible fraction of the light that P_1 receives from the interior to get sufficiently deviated in the region surrounding P_1, so as to proceed toward the earth along the tangent P_1E. The observer will now consider P_1 to belong to the solar disk.

The transition from disk to surroundings will appear abrupt if the minimum distance between levels like P and levels like Q be

less than 700 kilometers (one second of arc). This condition is compatible with a rather slow radial pressure gradient, because it only requires that the average radius of curvature[1] ($\rho = n \div \frac{dn}{ds}$) of rays deviated by irregular gradients of optical density be about 9 times greater in Q than in P. (Even a smaller ratio would probably suffice.) There will then appear a circular boundary between P and Q, lying in a plane through the sun's center perpendicular to the line of sight, but there is no particular "solar surface" corresponding to it.[2]

In a level P just inside the apparent photosphere the average value of ρ may still be of the order of magnitude 10^{10} cm. We can easily show that to such curvatures of rays quite reasonable density gradients correspond. For if we suppose hydrogen to be a principal constituent of the visible layers, the average refraction-constant $R = n - 1/\Delta$ of the medium may be estimated at 1.5. Putting this value, and $\rho = 10^{10}$, into the relation

$$\frac{d\Delta}{ds} = \frac{1}{R\rho}^{*}$$

* Cf. *Astrophysical Journal*, **25**, 107, 1907.

we obtain the density-gradient 6×10^{-11}, which means that in two points one kilometer (10^5 cm) distant from each other the density

[1] "Average radius of curvature" is here used as an abbreviated expression for "the radius of curvature corresponding to the average value of that radial component of the irregular density gradients, which is directed toward the center of the sun."

[2] At first sight one might be inclined to think that the boundary thus defined has the same radius as Schmidt's critical sphere would have. On closer examination, however, the two notions appear to be entirely different. This is clearly brought out with the aid of the following analogous conception. Imagine a spherical mass of liquid (radius R) of constant *average* optical density, and, as a source of light in the middle of it, an incandescent lamp provided with a big globe of milky glass (radius $\frac{1}{2}$ R). As there is no radial density-gradient, a critical sphere in the sense of Schmidt's theory could not appear in that medium. Let the liquid be a mixture of a solution of common salt and a solution of glycerine in water, both solutions having the same specific weight but different refracting power (cf. *Physik. Zeitschr.* **11**, 59, 1910). If we now suppose that only in the outer spherical shell (radii R and $\frac{3}{4}$ R) the solutions are completely mixed, whereas in the inner shell surrounding the luminous globe the liquids are only stirred, but still honeycombed with irregular gradients of optical density—the *average* optical density of the shells being the same—then the inner shell will seem to be a self-luminous body. The origin of its boundary is comparable with that of the solar limb according to our theory.

The above interpretation of the photosphere evidently involves an explanation of the reversing layer and the chromosphere as soon as we take account of anomalous dispersion. On this subject, however, we shall not expatiate in the present paper.

only differs 0.000006, i.e., 0.5 per cent of the density of our terrestrial atmosphere. It would be very remarkable, indeed, if the general circulation in the sun did not bring along local differences of temperature and of composition sufficient to account for density-gradients of that order of magnitude. In a layer, for instance, where the average density does not exceed the density of our own atmosphere at sea-level, a temperature gradient of 1°4 C. per kilometer is all that would be required.

The above dioptrical conception of the photosphere implies the following explanation of the variation of brightness across the disk.

This problem, indeed, may also be expressed as follows: What is the cause of the fact that the *irradiation surface* of a point M,

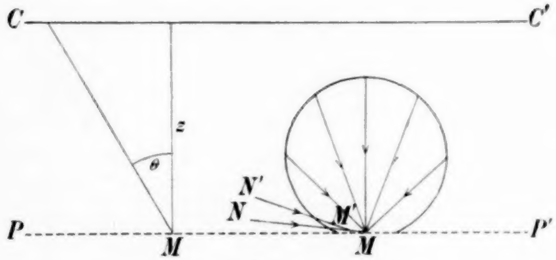

Fig. 3

lying somewhere in or near the "photospheric level," has that particular shape (different according to the selected wave-length) which direct observation assigns to it?

Let PP' (Fig. 3) represent a part of the photospheric level, CC' of another level lying so much deeper that there the solar matter is dense enough to emit light giving a continuous spectrum.

Although the medium surrounding M be a mixture of selectively absorbing gases, transparent to the greater part of the spectrum, that transparency is not absolute. *Molecular scattering* (Rayleigh)[1] weakens a *direct* beam according to the law $I=I_0e^{-sz}$, in which $s=\frac{32\pi^3(n-1)^2}{3\lambda^4N}$; but if the source of light be an incandescent surface CC' radiating the energy I_0 per square unit, and

[1] Rayleigh., *Phil. Mag.* (5), **47**, 375, 1899.

if the diffused light itself be taken into consideration, the energy emerging per square unit from PP' will (as found by Schuster[1]) be expressed thus:

$$I = I_0 \frac{2}{2+sz}.$$

We are aware that this formula does not hold exactly for non-homogeneous media, nor for oblique directions when simply replacing z by $z \sec \theta$; but as a first approximation we shall put

$$J = J_0 \frac{2}{2+s\,z \sec \theta},$$

where J and J_0 now refer to units of surface located in the layers PP' and CC' respectively, and taken perpendicular to the direction considered. Supposing J_0 to be independent of direction, we find that J decreases as θ increases, in agreement with the characteristic of the irradiation surface.[2]

One of the reasons why the latter equation cannot be expected to represent the conditions completely is that it does not allow for possible incurvation of the direct beams passing through the medium. If θ approaches the value 90°, our formula makes J tend toward zero, whereas in reality the brightness at the limb only falls to values between 0.13 $J_{\theta=0}$ and 0.30 $J_{\theta=0}$ with different colors. Now, it is evident that refraction by the irregular density-gradients at once accounts for the discrepancy; indeed, a beam reaching M along NM (θ nearly$=90°$) might have been turned into that direction from another direction $N'M'$ for which θ has a smaller value, so that J will have a greater value than the one corresponding to the formula. It is exactly this process on which our explanation of the sun's edge was based.

[1] Schuster, *Astrophysical Journal*, **21**, 1, 1905. Abbot, in his valuable book *The Sun* (1911), also introduces molecular scattering as a principal agent in producing the appearance of the photosphere.

[2] A full comparison of the theoretical with the observational irradiation surfaces for different wave-lengths will be given at a later date. If 2 may be neglected as compared with $sz \sec \theta$, the expression for J becomes $J = J_0 \frac{2}{sz} \cdot \cos \theta$, which represents a sphere, tangent to the photospheric level in M. The irradiation surface, as constructed with the values for violet light taken from H. C. Vogel's well-known table (*Ber. der Berl. Akad.*, 1877, p. 104), is in its main part strikingly similar to such a sphere.

If, therefore, we consider both scattering and irregular refraction effects, the conclusions to which the theory leads are compatible with the observed shape of the irradiation surface, or with the distribution of the intensity on the solar disk.

The agreement also prevails when kinds of light of different wave-lengths are considered. Let us distinguish between, e.g., red and violet, by introducing the subscripts r and v.

At the center of the disk ($\theta=0$) we have between the intensities of red and violet light the proportion

$$p_0=\frac{J_r}{J_v}=\frac{J_{0,r}}{J_{0,v}}\cdot\frac{2+s_v z}{2+s_r z},$$

in which, according to Rayleigh's formula, $s_v>s_r$ (if cases of anomalous dispersion be excluded, so that the disparity between n_v and n_r may be neglected).

At a point, corresponding to the angle θ, we have

$$p_\theta=\frac{J_{0,r}}{J_{0,v}}\cdot\frac{2+s_v z\sec\theta}{2+s_r z\sec\theta}.$$

The second factor of p_0 is greater than unity, and p_θ is greater than p_0. This means that the longer waves preponderate as we proceed from the center of the disk outward. With increasing values of sec θ, $p\theta$ approaches the limit

$$p_{90°}=\frac{J_{0,r}}{J_{0,v}}\cdot\frac{s_v}{s_r}=\frac{J_{0,r}}{J_{0\,v}}\cdot\frac{\lambda_r^4}{\lambda_v^4};$$

this proportion, however, will be more or less modified by irregular refraction.

Taking all in all, the above theory of the photosphere thus appears to account for the sun's edge, and for the principal features of the results of Vogel's well-known spectrophotometric measurements.

It implies at the same time an interpretation of the *granular structure* of the solar disk. If Anderson[1] and other astrophysicists were right in assuming the irradiation surface of a point M near the photospheric level to be a hemisphere (sps_1 in Fig. 1, p. 131), irregular gradients of optical density could not produce any sensible dis-

[1] *Astrophysical Journal*, **31**, 166, 1910.

turbance in the uniform brightness of the disk, except in special cases. But their assumption certainly is erroneous; the average intensity of the light passing through M varies considerably with the value of the angle θ; so the irregular refraction of the light must result necessarily in variegation of luminosity.

Waves that undergo anomalous refraction will of course deviate to a higher degree in the same gradients. Following out this line of thought, we arrive at explanations of spectroheliograph results,[1] on which we shall not now insist.

A few remarks may be added in connection with the sun-spot hypothesis suggested in 1909.[2] A spot was supposed to be a region where, from a central minimum outward, the optical density increases with a gradually decreasing gradient. If sun-spots are solar vortices, such conditions are very likely to obtain. It was then argued that, when a similar structure is traversed by the light from an extensive source radiating, as the photosphere does, with intensities decreasing from the center toward the limb, refraction must exactly produce the characteristic optical features observed in a spot: an umbra surrounded by a penumbra. Taking anomalous dispersion effects into consideration, one is led by the same argument to an explanation of the principal properties of the spot-spectrum. Lately we succeeded in realizing, in the laboratory, the formation of a typical "sun-spot" by refraction of light in a whirling mass of gas, and could witness several phenomena rather closely resembling the appearances produced by the real solar objects. A description of those experiments, together with a discussion of their possible bearing on several spot-problems (e.g., on the apparent effect of the earth on the formation and growth of sun-spots), must be deferred to a separate paper.

We now only wish to emphasize that the above conception of sun-spots naturally fits in with our dioptrical explanation of the photosphere. The levels where vortex-motion should occur so as to produce the appearance of a spot will be found somewhere between spheres corresponding to PP and QQ of our Fig. 2. The conditions in a spot need not differ very much from those obtaining

[1] Cf. *Astrophysical Journal*, **21**, 278, 1905; **28**, 360, 1908; **31**, 419, 1910.

[2] *Proc. Roy .Acad. Amst.*, **12**, 273, 1909; *Physik. Zeitschr.*, **11**, 62, 1910.

in the surrounding regions. Their chief characteristics are: (1) the rotary motion, which determines a magnetic field and a systematic arrangement of density-gradients (that need not be steeper than the average irregular gradients otherwise present in the same levels), and (2) the differences of temperature and of composition connected with the special form of circulation.

SUMMARY

Various views concerning the nature of the photosphere are criticized, and a new dioptrical interpretation of several photospheric phenomena is proposed.

UTRECHT
May 1913

A FURTHER CONTRIBUTION TOWARD THE ESTABLISHMENT OF A NORMAL SYSTEM OF WAVE-LENGTHS IN THE ARC SPECTRUM OF IRON

By F. GOOS

1. *The aim of this article* is to point out again that for the establishment of a normal system of wave-lengths in the arc spectrum of iron, like that which has been adopted by the International Union for Solar Research, it is not sufficient to prescribe a current of 5 to 10 amperes[1] for the arc, but is absolutely necessary to define exactly the manner of burning and the part of the arc used.

By way of proof, the difference in the values of the three observers of the normals of the second order will be pointed out; the wave-lengths of the iron arc already published by Kayser[2] and by myself[3] will be compared with the measurements of St. John and Ware[4] which have recently appeared; and measurements of the widths of some selected iron lines will be given. Further, in conclusion, I shall offer suggestions as to an iron arc which will be as satisfactory as possible.

On the basis of their measurements made in Pasadena and on Mount Wilson, where the difference in altitude corresponds to a difference in barometric height of about $\frac{1}{5}$ atmosphere, St. John and Ware conclude that in the choice of standard lines, the extent to which the lines are sensitive to pressure must be carefully investigated, since especially in the less refrangible part of the spectrum the pressure-shifts reach values which cannot be ignored in observatories at high altitudes.

My observations with different sorts of iron arcs now show that, with the same external pressure—atmospheric pressure—on plates from different parts of one and the same arc, further by

[1] *Trans. Internat. Union for Solar Research*, **1**, 238, 1906.

[2] *Zeitschr. f. wiss. Phot.*, **9**, 173, 1911.

[3] *Ibid.*, **11**, 1, 1912; *Astrophysical Journal*, **35**, 221, 1912; *Zeitschr. f. wiss. Phot.*, **11**, 305, 1912; *Astrophysical Journal*, **37**, 48, 1913.

[4] *Astrophysical Journal*, **36**, 14, 1912.

change of current, arc length, etc., line-shifts occur which are of the same order of magnitude as those observed by St. John and Ware. I would like to offer as a hypothesis at this point that these shifts are due in part to pressure differences; that higher pressures prevail at the negative pole than in the middle of the arc and at the positive pole; and that with larger currents the pressure within the arc increases. This is because with larger currents more iron vaporizes, the vapor-density becomes greater, and a pressure arises in the inner parts of the arc.

2. *Normals of the second order.*—St. John and Ware in their work come to the conclusion that the international normals of the second order form a homogeneous system to within 0.001 Å. This is perhaps the case, but it seems strange that the values of the three different observers, the means of which form the international values, show systematic differences with respect to each other of several thousandths of an Ångström unit.

I believe that these differences are due to the fact that the three observers worked with different sorts of iron arcs. Fabry and Buisson used iron rods 7 mm thick, 3 to 5 amperes, and 110 or 220 volts; Eversheim 8 mm rods, 5 amperes, and 220 volts, while Pfund probably used the arc described by him,[1] with a small iron sphere as the positive electrode, 3.5 amperes and 220 volts. Eversheim, then, worked with larger currents than Fabry and Buisson and Pfund. But the larger current probably caused a higher pressure and in general shifted the lines toward the red. If now we make use of the observations of Gale and Adams[2] on the pressure-shifts in the iron spectrum, we see that all lines do not suffer the same shift, but that they may be divided into several groups of different susceptibility to pressure.

In Table I are collected, for the normals of the second order, the values of Fabry and Buisson, Eversheim, and Pfund;[3] the classification of Gale and Adams together with the shift for an increase in pressure of 8 atmospheres (Δ), and the differences, Eversheim−Fabry and Buisson, and Eversheim−Pfund.

[1] *Zeitschr. f. wiss. Phot.*, **6**, 326, 1908.

[2] *Astrophysical Journal*, **35**, 10, 1912.

[3] *Ibid.*, **32**, 215, 1910; **33**, 85, 1911.

TABLE I

λ	Fabry and Buisson	Eversheim	Pfund	Group	Δ (8 Atm.)	Eversheim — F. and B.	Eversheim — Pfund
	I.Å.	I.Å.	I.Å.		Å.	Å.	Å.
5371	.498	.493	.494		0.029	−0.005	−0.001
5405	.780	.780	.780		27	0	0
34	.530	.524	.528		27	− 6	− 4
55	.616	.611	.614	*a*	29	− 5	− 3
97	.521	.523	.523		30	+ 2	0
5506	.783	.785	.784		0.031	+ 2	+ 1
6027	.059	....	.059		0.062	....	....
65	.493	.493	.491		77	0	+ 2
6137	.700	....	.702		78	....	....
91	.569	.568	.567		86	− 1	+ 1
6230	.732	.736	.735		70	+ 4	+ 1
65	.147	....	.143	*b*	70	....	....
6318	.029	.028	.026		80	− 1	+ 2
35	.343	.342	.337		74	− 1	+ 5
93	.612	.613	.612		72	+ 1	+ 1
6430	.859	.862	.855		68	+ 3	+ 7
94	.994	.994	.992		0.065	0	+ 2
5232	.958	.958	.956		0.11	0	+ 2
66	.568	.569	.569		13	+ 1	0
5324	.196	.196	.195		12	0	+ 1
5569	.632	.636	.631	*d*	14	+ 4	+ 5
86	.770	.773	.772		12	+ 3	+ 1
5615	.658	.662	.663		13	+ 4	− 1
58	.835	.838	.835		0.15	+ 3	+ 3

If we take the means of the three groups, we get:

	Eversheim — F. and B.	Eversheim — Pfund	Δ (8 Atm.)
Group *a*	−0.0020 Å.	−0.0012 Å.	0.029 Å.
b	+0.0006	+0.0026	0.073
d	+0.0021	+0.0016	0.125

Even though these numerical values cannot be regarded as at all reliable, as a glance at the separate values of the differences Eversheim — Fabry and Buisson and Eversheim — Pfund shows, nevertheless, I believe that the signs at least are correct; that is, that for the two groups *b* and *d*, which are shifted to the red by pressure more strongly than group *a*, the wave-lengths of Eversheim are relatively larger than those of Fabry and Buisson and of Pfund, corresponding to the larger current with which Eversheim operated his arc. In the case of group *a*, the sign ought

really to be positive since there is a small shift toward the red in the case of this group also. The negative sign indicates that other systematic differences exist among the observers in addition to the pressure differences, which are to be sought for perhaps in the measurement itself, perhaps in a comparison of the light sources. But even group *a* is not very satisfactory in the agreement of the separate values, Eversheim—Fabry and Buisson, so that no definite conclusions may be drawn from it. As I shall point out later, the differences in wave-length, which are found in arcs burning very differently, correspond to a pressure difference of about $\frac{1}{3}$ of an atmosphere. I would estimate the pressure difference between the arc of Eversheim and that of Fabry and Buisson, and of Pfund to be $\frac{1}{8}$ atmosphere at most. Now Gale and Adams showed that at 8 atmospheres the lines of group *d* suffer a shift of 0.096 Å., relative to the lines of group *a;* that is, a shift of 0.0024 Å. for a pressure of $\frac{1}{8}$ atmosphere (in case the law of proportionality holds). We arrive thus at values which agree well in order of magnitude with those found above. In spite of the differences among the three separate observers, each of the three wave-length systems forms for itself a homogeneous system with respect to each of the iron arcs which was used. Thus the mean of the three, the system of international normals of the second order, forms a homogeneous system with respect to the iron arc whose properties are a mean of those used by the three observers. For any other arc, for example, one carrying 6 to 10 amperes, the system will no longer be homogeneous.

If, now, normals of the third order are derived from normals of the second order, the greatest possible care must be taken to use always exactly the same light-source from which the normals of the second order were obtained; for in the iron spectrum there are many lines even more sensitive to pressure than those listed above, but which cannot be well dispensed with as normals of the third order (contrary to the opinion of St. John and Ware, who would exclude these lines). Otherwise there exist wave-length differences, as the measurements of Kayser, of St. John and Ware, and of myself show both in comparison with one another, and with the normals of the second order.

Naturally, it is not easy to specify afterward for just what arc the International System of the second order holds, but I believe that it would be an iron arc carrying about 4 amperes, drawn out moderately long (about 5 mm) and of which only the central part was used. At any rate, the outstanding discrepancies among normals of the second order will scarcely amount to 0.001 Å. with this arc. But this degree of exactness may just about be obtained for the best lines, in determining the normals of the third order with gratings.

3. *Normals of the third order.*—At the time when I published my measurements in the less refrangible part of the spectrum, it was not possible to compare my wave-lengths with Kayser's in an entirely satisfactory way, since I could not isolate the systematic errors in Kayser's values, in order to investigate the outstanding errors, the source of which was unknown to me at that time. I believe that it has now become possible to do this. I will at any rate again assume in what follows, that differences in pressure exist in different sorts of arcs which are in the main responsible for the wave-length differences. Now there are, as we see from the measurements of Gale and Adams, and St. John and Ware, two large regions in the green and red where all the normals of the second order belong to the same group, with respect to pressure-shift, and where in addition several other lines are present which have been measured as normals of the third order by Kayser, by St. John and Ware, and by myself. There are from λ 5371 to λ 5535, 10 lines of group *a*, and from λ 6027 and λ 6494, 21 lines of group *b*. In addition, there is a third such region between λ 5569 and λ5658 to which the lines of sub-group *d* (according to St. John and Ware) belong. This group seems to me to be not entirely free from objection (as indeed the differences Pasadena — Mount Wilson show) since its susceptibility to pressure is considerable while that of groups *a* and *b* is very much less.

Within the limits of groups *a* and *b* there should, then, be no systematic differences among the observers, Kayser, St. John and Ware, and myself, or if they exist, they are attributable to another cause. Table II gives the differences Pasadena — Kayser, and Pasadena — Goos, for groups *a* and *b*.

As we see, the differences Pasadena — Goos show a small systematic

range, for which I can offer no explanation at this time. The differences Pasadena—Kayser are somewhat larger and are perhaps to be attributed to Kayser's systematic errors, mentioned previously[1] by me. A graphical adjustment yields the following corrections (Table III) which must be applied to Kayser's values and to mine to make them comparable with the Pasadena values.

TABLE II

λ	Group	Pasadena −Kayser	Pasadena −Goos	Δ 8 Atm.
I.Å.		Å.	Å.	Å.
5371.5	a	+0.005	0.000	0.029
5405.8	a	+ 2	0	27
29.7	a	+ 1	+ 2	29
34.5	a	+ 2	+ 3	27
46.9	a	+ 1	− 5	31
55.6	a	− 2	....	29
97.5	a	+ 1	0	30
5501.5	a	0	− 4	30
06.8	a	+ 1	0	31
35.4	a	− 1	0	34
			Mean	0.030
		For λ 5371–λ 5429	mean	0.028

λ	Group	Pasadena −Kayser	Pasadena −Goos	Δ 8 Atm.
I.Å.		Å.	Å.	Å.
6027.1	b	−0.003	+0.002	0.062
65.5	b	+ 3	0	77
6136.6	b	− 2	+ 4	82
37.7	b	− 2	0	78
57.7	b	− 9	− 5	41
73.3	b	− 9	− 6	67
91.6	b	0	− 1	86
6200.3	b	− 8	− 2	79
13.4	b	− 1	− 10	72
19.3	b	− 2	− 7	73
30.7	b	0	+ 2	70
52.6	b	+ 2	+ 3	77
54.3	b	− 2	− 4	64
65.1	b	0	− 1	70
97.8	b	− 4	− 3	68
6318.0	b	− 3	0	80
35.3	b	+ 2	0	74
93.6	b	0	+ 1	72
6421.4	b	+ 9	− 3	68
30.9	b	+ 11	− 1	68
95.0	b	− 1	+ 1	65
			Mean	0.071
		For λ 6027–λ 6136	mean	0.074
		For λ 6136–λ 6421	mean	0.072

After applying these corrections, let us compare the Pasadena values with Kayser's and with mine for a few other lines within the limits mentioned above. There are between λ 6147 and λ 6411, 11 lines of group *d* which lie between normals of the second order belonging to group *b*. Moreover between λ 5383 and λ 5424, there are 4 lines of group *e* (shifted to the violet by pressure, according to St. John and Ware) which lie between normals belonging to

[1] *Zeitschr. f. wiss. Phot.*, **10**, 200, 1911.

group *a;* and between λ 6042 and λ 6078, there are three lines of group *e* which lie between normals belonging to group *b*.

TABLE III

From λ to λ	Correction Kayser	From λ to λ	Correction Goos
	Å.		Å.
5370–5400	+0.003	5370–5460	0.000
5400–5420	+ 2	5460–5540	− 1
5420–5450	+ 1		
5450–5540	0		
6020–6090	0	6020–6040	+ 1
6090–6110	− 1	6040–6080	0
6110–6130	− 2	6080–6110	− 1
6130–6140	− 3	6110–6320	− 2
6140–6160	− 4	6320–6410	− 1
6160–6180	− 5	6410–6500	0
6180–6200	− 4		
6200–6230	− 3		
6230–6260	− 2		
6260–6330	− 1		
6330–6360	0		
6360–6380	+ 1		
6380–6410	+ 2		
6410–	+ 3		

Table IV gives the required data. Under Δ are the pressure-shifts of Gale and Adams for 8 atmospheres, under Δ_1, the pressure-shift for $\frac{1}{5}$ atmosphere difference of pressure as found by St. John and Ware, through their measurements in Pasadena and on Mount Wilson.

The differences Pasadena−Kayser are in general positive. The mean of the 11 values is +0.001 Å. The value −0.012 for λ 6400 is quite exceptional, and if it were omitted, the mean would be +0.002 Å. The differences Pasadena−Goos are very irregular, but in general the negative values are larger than the positive ones, and the mean is −0.002 Å. If these differences are regarded as due to pressure-shifts, it means that Kayser worked with an arc of lower internal pressure than the Pasadena observers, and Goos, on the contrary, with one of higher pressure. The differences Pasadena−Goos especially, on account of their large variation, do not seem to me to be sufficiently well explained on the ground of

pressure alone. The accidental errors are not so large, as is shown by Table II for the lines of the *b* group in this spectral region. We must therefore assume that there are other factors which can influence the position of these lines.

TABLE IV

λ	Pasadena	Kayser	Goos	Pasadena − Kayser	Pasadena − Goos	Group	Δ For 8 Atm.	Δ_1 for ½ Atm.
	I.Å.	I.Å.	I.Å.	Å.	Å.		Å.	Å.
6147	.844	.840	.844	+0.004	0.000			+0.008
51	.636	.629	.631	+ 7	+ 5			+ 8
80	.225	.226	.216	− 1	+ 9			+ 9
6232	.669	.667	.671	+ 2	− 2			+ 7
46	.350	.343	.349	+ 7	+ 1	*d* Shifted to the red by pressure	0.28	+ 13
6301	.531	.527	.529	+ 4	+ 2		0.25	+ 13
02	.520	.521	.523	− 1	− 3			+ 9
36	.851	.850	.854	+ 1	− 3		0.26	+ 8
6400	.026	.038	.035	(− 12)	− 9		0.24	+ 6
08	.044	.047	.057	− 3	− 13			+ 6
11	.678	.676	.685	+ 2	− 7		2.23	+ 10
			Mean	+0.001	−0.002		0.25	+0.009
	λ 6400 omitted		mean	+0.002				
5383	.353	.363	.353	− 10	0			− 18
5410	.890	.904	.878	− 14	+ 13			− 21
15	.175	.186	.170	− 11	+ 5			− 20
24	.038	.051	.033	− 13	+ 5			− 22
			Mean	−0.012	+0.006	*e* Shifted to the violet by pressure	Unmeasurable Mean	−0.020
6042	.083	.092	.084	− 9	− 1			− 10
55	.983	.992	.980	− 9	+ 3			− 12
78	.470	.476	.466	− 6	+ 4			− 12
			Mean	−0.008	+0.002		Mean	−0.011

The situation is easier to interpret for the lines of the *e* group. For four lines in the green the mean Pasadena−Kayser is −0.012 Å., and Pasadena−Goos is +0.006 Å. For three lines in the red the corresponding means are −0.008 Å. and +0.002 Å. There can be no doubt of a decided systematic difference here. Regarded as pressure-shifts, they indicate as above that lower pressures existed in Kayser's arc, and higher pressures in mine than in the arc used by St. John and Ware. This is also in harmony with the data of the observers. St. John and Ware worked with the Pfund arc, at 6 amperes and 110 volts, and thus in any

case, of moderate length; Kayser used about the same currents, always with a very long arc, in order to be able to cover the long slit of his concave grating. I myself used an arc at 6 or 7 amperes, taken especially short, since it then burns very quietly and with great brilliancy. All observations (I shall take this up again later) point to the conclusion that in the middle portion of a long arc, with its sharp fine lines, the pressure is less than in a short arc which shows diffuse and greatly broadened lines. If we compute the amount in atmospheres by which the pressure is greater than and less than the Pasadena pressures, from the systematic line-shifts of groups *d* and *e*, with the aid of the values Δ and Δ_1, we get the following table. (It should be noted that $\Delta=0.25$ Å. is the absolute shift for a pressure of 8 atmospheres, while we are dealing here with the relative shifts with respect to the normals of the second order which belong to the group *b*, $\Delta=0.072$.)

TABLE V

Group	Kayser	Goos
d	From Δ decrease in pressure $\frac{1}{11}$ Atm.	From Δ increase in pressure $\frac{1}{11}$ Atm.
d	From Δ_1 decrease in pressure $\frac{1}{22}$ Atm.	From Δ_1 increase in pressure $\frac{1}{22}$ Atm.
e green	From Δ_1 decrease in pressure $\frac{1}{8}$ Atm.	From Δ_1 increase in pressure $\frac{1}{17}$ Atm.
e red	From Δ_1 decrease in pressure $\frac{1}{7}$ Atm.	From Δ_1 increase in pressure $\frac{1}{28}$ Atm.

The value $\frac{1}{11}$ atmosphere deduced for group *d* from Δ is in any case not free from objection and is too large, since many of the lines of this group could not be measured by Gale and Adams at 8 atmospheres pressure and showed a greater susceptibility to pressure than corresponds to the accepted mean value of 0.25 Å. The value deduced for the three groups, *d*, *e* green, and *e* red, from Δ_1 agree fairly well. The means give for Kayser a diminution of pressure of $\frac{1}{10}$ atmosphere, and for Goos an increase in pressure of $\frac{1}{21}$ atmosphere as compared with Pasadena.

The assertion above, that in the middle portion of a long arc the pressure is less than in a short arc, I shall support by two series of measurements (which were published in my last article). Plane grating measurements on an arc 3 mm long were compared with Fabry-Perot interferometer measurements on the middle part of

an arc 10 mm long; further, concave grating measurements, which I was able to make with Kayser's large grating during the course of a visit at Bonn, on an arc 3 to 4 mm long were compared with measurements on an arc 8 to 9 mm long. The current was, in all cases, 6 to 7 amperes.

TABLE VI

λ	Arc 3 mm	Arc 10 mm	Diff.	Group	λ	Arc 3–4 mm	Arc 8–9 mm	Diff.	Group
	I.Å.	I.Å.	Å.			I.Å.	I.Å.	Å.	
5371	.495	.498	+0.003	*a*	5554	.872	.893	+0.021	*e*
5410	.878	.915	+ 37	*e*	65	.689	.704	+ 15	*e*
15	.170	.203	+ 33	*e*	69	.632	.632	0	*d*
24	.033	.066	+ 33	*e*	72	.852	.856	+ 4	*d*
34	.526	.529	+ 3	*a*	76	.100	.104	+ 4	*d*
97	.522	.518	− 4	*a*	86	.773	.772	− 1	*d*
					98	.288	.307	+ 19	*e*
					5602	.961	.964	+ 3	*d*
					15	.659	.660	+ 1	*d*
					24	.559	.558	− 1	*d*
					38	.279	.272	− 7	*d*
					58	.837	.836	− 1	*d*

In the region from λ 5371 to λ 5497, the normals of the second order which served as reference lines belonged to group *a*, the other three lines to group *e*. These show a shift 0.034 Å. to the violet in the middle of the 3 mm arc, which according to St. John and Ware corresponds to an increase in pressure of $\frac{1}{3}$ atmosphere. In the region from λ 5554 to λ 5658, the lines λ 5569, λ 5586, λ 5615, and λ 5658, which belong to group *d*, were used as reference lines. Of the remaining 8 lines, 5 belonged to group *d* and should show no pressure-shift (in the mean, the difference for these lines is −0.001 Å.). The three remaining lines, which belong to group *e*, on the contrary, show in the mean a shift of 0.018 Å. to the violet, which corresponds to an excess of pressure in the short arc, compared to the long arc, of $\frac{1}{5}$ atmosphere.

4. *Breadth of lines.*—Another phenomenon is closely allied to pressure-shift, namely, the broadening of the lines. It is to the point, therefore, to measure the width of lines in different parts of the arc, and at different current strengths. A Fabry-Perot interferometer, with adjustable distance between the plates, is convenient for this,

and I have constructed one of simple form[1] for this purpose. The more homogeneous the radiation, the clearer the fringes, and they become weaker with increasing distance between the plates, and finally disappear for a certain difference of path, D, equal to twice the distance between the interferometer plates. From this limiting distance D one may get a measure of the width of the lines by the relation $d\lambda=\frac{\lambda^2}{D}$ (Fabry and Buisson).[2] The width of the line would be given directly by $d\lambda$, if the line had sharp edges, i.e., if it were a sharply defined region from λ to $\lambda+d\lambda$, and in case the apparatus were perfect. In spite of the fact that these conditions are not fulfilled, and that we are dealing in part with very diffuse lines, which one can hardly regard as having a definite width, I shall, in what follows, call $d\lambda$ the width of the lines.

Table VII refers to an arc 5 mm long, between two iron poles 7 mm in diameter, and carrying a current of 5.6 amperes. The potential of the current was 220 volts, the fall of potential across the arc, 49 volts. The negative pole was above. I have determined, visually, the width of 25 lines in five different zones of this arc. For the finest lines, a path difference, D, of more than 45 mm was required, before the interference fringes disappeared.

The separate zones are circular surfaces 1 mm in diameter. Zone 1 lies immediately at the negative pole, zone 3 in the middle of the arc, and zone 5 at the positive pole.

The lines are again arranged in groups according to their susceptibility to pressure. An increase of width at the negative pole is shown for all the lines. It appears also that with groups d and e there is a slight increase at the positive pole, but the observations here are difficult on account of the diminished brightness of the lines. Immediately at the negative pole, the mean width of the lines of group a is 0.07 Å., of group d 0.14 Å., of group e 0.33 Å. The pressure-shift per atmosphere for these groups is +0.004 Å., +0.02 Å., and −0.06 Å.

I have also determined the width of a few very sensitive lines which apparently all belong to group e, for different current

[1] *Zeitschr. f. Instrumentenkunde*, **32**, 326, 1912.

[2] *Astrophysical Journal*, **31**, 115, 1910.

strengths, using a very short arc, with the iron pole-pieces separated only 1 mm. The results are collected in Table VIII.

I used again a 220-volt circuit. The potential difference at the electrodes is given in the table. For currents of 4.5 and 6 amperes I have given two sets of observations, one with iron poles

TABLE VII

λ	−Pole ←««« Zone »»→ Pole+					Group
	1	2	3	4	5	
I.Å.	Å.	Å.	Å.	Å.	Å.	
5341.0	0.06	0.06	0.06	0.06		
71.5	9	6	6	6		
97.1	9	9	6	< 6		
5405.8	6	6	6	6	<0.06	
29.7	9	6	< 6	< 6	< 6	*a*
34.5	6	6	< 6	< 6	< 6	
46.9	9	6	< 6	< 6	< 6	
97.5	6	< 6	< 6			
5501.5	6	6	< 6			
06.8	6	< 6	< 6			
Mean	0.07	0.06	0.05	0.05	0.05	
5324.2	13	6	6	6	9	
39.9	9	6	6	6		
93.2	13	6	9	6	9	
5569.6	10	7	7			
72.8	10	7	7			
76.1	14	7				*d*
86.8	14	7	< 7	7	< 10	
5603.0	14	10				
15.7	14	10	< 7	7	< 10	
24.6	17	10				
58.8	17	< 10				
Mean	0.14	0.08	0.07	0.06	0.09	
5383.4	25	18	13	13	18	
5410.9	36	13	13	13	18	*e*
15.2	36	26	13	13	18	
24.0	36	26	18	18	18	
Mean	0.33	0.21	0.14	0.14	0.18	

5 mm and one with iron poles 9 mm in diameter. As we see, at 4.5 amperes the width of the line is somewhat less for the 9 mm poles than for the 5 mm poles; but using a current of 6 amperes there is scarcely any difference. It was not possible to get a satisfactory arc with 3 amperes and 9 mm poles or with 9 amperes and 5 mm

poles, in the first case on account of the weakening of the light, and in the second on account of the melting of the iron. In the mean we get for 3, 4.5, 6, and 9 amperes line widths of 0.31, 0.38, 0.46,

TABLE VIII

λ	3 Amp. 36 Volts 5 mm	4.5 Amp. 33 Volts 5 mm	4.5 Amp. 34 Volts 9 mm	6 Amp. 32 Volts 5 mm	6 Amp. 33 Volts 9 mm	9 Amp. 31 Volts 9 mm
I.Å.	Å.	Å.	Å.	Å.	Å.	Å.
5367.4	0.32	0.48	0.36	0.48	0.48	0.71
69.9	32	48	36	48	48	71
83.4	29	36	29	48	48	71
5404.0	27	32	26	36	42	59
10.9	29	42	33	45	42	59
15.2	33	42	42	49	49	72
24.0	37	49	42	49	49	72
Mean	0.31	0.42	0.35	0.46	0.47	0.68

and 0.68 Å. This relation between width of line and current may be expressed very well by the equation

$$d\lambda=0.28+0.005\ i^2\ .$$

The breadth $d\lambda$, computed by this formula for the current strengths i, are 0.32, 0.38, 0.46, and 0.68 Å. This formula also shows that the width is a function of the square of the current strength, and that for lines of group e, with an arc 1 mm long, even with very small currents, the average width $d\lambda$ does not fall below 0.28 Å.

TABLE IX

λ	9 Amp. 40 Volts, 9 mm Poles		
	−Pole	Center	+Pole
I.A.	Å.	Å.	Å.
5367.4	0.48	0.29	0.32
69.9	48	32	32
83.4	41	26	24
5404.0	37	27	24
10.9	37	27	24
15.2	42	33	29
24.0	49	37	33
Mean	0.43	0.30	0.28

In conclusion, the widths of the same lines are given in Table IX for three zones (at the negative pole, in the middle and at the posi-

tive pole,) using an arc 3 mm long and a current of 9 amperes between iron poles 9 mm in diameter.

We see that even immediately at the negative pole itself the width is not nearly so great as in the arc 1 mm long.

The whole investigation shows us again that the iron arc is not homogeneous. Groups of lines exist which change their widths in different ways, both within one and the same arc, and when arc length and current strength are changed. These groups are the same which are concerned in pressure susceptibility. Within the different sorts of arcs, pressure differences exist which produce the same effect as increased or diminished external pressure.

5. *Suggestions for the determination of normals of the third order.*— All of the measurements of normals of the third order which have been published up to the present time show the inadequacy of the specifications for the iron arc. The iron arc is not homogeneous in itself; radiations from different parts of the arc give different results, and length of arc and current strength are influential to a high degree. First of all, the lack of symmetry of the arc is very disturbing. While the conditions appear to change but little from the middle of the arc to the positive pole, a very significant broadening of the lines is perceived from the middle of the arc toward the negative pole. I believe that this lack of symmetry may best be rendered harmless by reversing the current, having the positive pole below for half the time, and the negative for the other half. It is very important also, that the arc burn quietly, be as bright as possible, but still furnish sufficiently sharp lines. On the basis of the many experiments which I have tried, and especially in view of what I have said in regard to the normals of the second order, I should like to propose that in the future for the normal spectrum of iron, an arc 5 mm long (separation of the rounded ends from each other) be used, between iron rods 6 mm in diameter, and with a current of 4 amperes. It should be used on a 220-volt circuit; the potential difference at the arc then falls to between 45 and 49 volts. It should be used with a pole changer, and the arc so projected on the slit of the spectrograph with the condensing lens that only a portion of the arc at the middle is used extending 1.5 mm vertically at most.

The difficulty of meeting this condition with the concave grating, as ordinarily mounted, should also be pointed out here. On account of the astigmatism it is only with difficulty that light from a prescribed portion of the arc can be isolated. Moreover, in consequence of the astigmatism, a long slit is required, which can be covered with the short middle portion of the arc only with the aid of a lens of very large angular aperture. It would perhaps be advantageous in work demanding the highest degree of exactness, to mount the concave grating "non-astigmatically," with the aid of a collimator (concave mirror) as Runge and Paschen,[1] and Fabry and Buisson[2] have already done. You get then, to be sure, only half the dispersion, though naturally the same resolving power, but in consequence you have on a plate of given length twice as many normals of the second order, making a better comparison possible, and have the great advantage of a fourfold light-intensity. The spectrum is normal within the same limits as in Rowland's mounting, but the constant varies somewhat in different parts of the spectrum.

TABLE X

	λ_o	λ_u	$\lambda_o-\lambda_u$	$\frac{\lambda_o+\lambda_u}{2}$	λ_m	$\lambda_m-\frac{\lambda_o+\lambda_u}{2}$	Group
	I.Å.	I.Å.	Å.	I.Å.	I.Å.	Å.	
5371	.492	.493	−0.001	.493	.495	+0.002	*a*
5405	.782	.782	0	.782	.780	− 2	*a*
10	.907	.900	+ 7	.903	.912	+ 9	*e*
15	.182	.182	0	.182	.190	+ 8	*e*
24	.050	.042	+ 8	.046	.056	+ 10	*e*
34	.525	.525	0	.525	.525	0	*a*
97	.524	.524	0	.524	.523	− 1	*a*

I have made some investigations on the homogeneity of the middle portion of an arc, like that described above, which burned excellently and was very bright. The lines of the *e* group are very good for this purpose; they are among the most sensitive in the whole spectrum. The whole length of the 5 mm arc was projected on the slit and afterward the spectrograms were measured at three different places: exactly in the middle (denoted in Table X by

[1] *Wied. Ann.*, **61**, 641, 1897.

[2] *Jour. de phys.*, IV, **9**, 929, 1910.

λ_m), 1.6 mm above the middle (λ_o), where the negative pole was during the first half of the exposure (three minutes) and the positive during the second half (three minutes); and finally, 1.6 mm below (λ_u).

As we see from the differences $\lambda_o - \lambda_u$, this arc also is not entirely symmetrical. This is because the bright flame which comes out from the negative pole, when the negative pole is below, stretches up in the form of a pointed flame, straight and long; when the negative pole is above, it is bent around by the hot-air streams and forced up. The differences $\lambda_m - \frac{\lambda_o + \lambda_u}{2}$ show the relative shifts of the three lines of group *e* with respect to the four lines of group *a* which served as standards. It is seen here also that the very sensitive lines of group *e* show systematic differences up to 0.01 Å., in moving 3.2 mm in a vertical direction. Within the previously prescribed limits of 1.5 mm, this would amount to only 0.003 Å. But it is now self-evident that the lines of group *e* must not be chosen as normals. With the other groups, particularly group *d*, the systematic differences will not amount to 0.002 Å.

In regard to the choice of normals of the third order, in opposition to the view of St. John and Ware, I am of the opinion that in addition to the lines of groups *a* and *b* (group *c* has no lines in this region) the lines of group *d* should also be included, since, without them, the intervals between the normals in many places become too large. It is naturally necessary then to reduce all observations to the normal atmospheric pressure of 760 mm, for which a knowledge of the pressure-shifts for a small reduction of pressure (in the range from about $\frac{1}{2}$ to 1 atmosphere) is necessary. Whether the measurements of the normals of the third order published up to the present time form a really homogeneous system of normals for the less refrangible part of the spectrum appears to me very questionable. The best means for attaining this end would be, in any case, to prepare an entirely new series of observations with more uniform light-sources.

An important question still remains open, namely, how best to fill in the gap in the unusable yellow-red part of the iron spectrum. I have made many investigations of nickel but have been unable

to find any arc which burns well, neither a combined iron-nickel arc nor one between rods of nickel-steel (25 and 36 per cent). But entirely aside from this point, the nickel lines in and of themselves are not satisfactory, and are apparently very sensitive to pressure, so that this metal on this account is not suitable to furnish normals. A light-source must be found which not only fills up the gap in the iron spectrum from λ 5700 to λ 5900, but which extends to some distance on each side, in order that it may not be necessary in the limiting regions, to work with two comparison light-sources. It must then be a light-source which possesses a larger number of suitable lines between λ 5500 and λ 6100.

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THE ORBITS OF EIGHTY-SEVEN ECLIPSING BINARIES—A SUMMARY

By HARLOW SHAPLEY

The results which are catalogued and briefly summarized in the present communication have been obtained from an extensive study of all accessible published and unpublished observations of eclipsing variables. More than a hundred thousand light-measures have been discussed in detail, representing the photometric work of thirty observers on the light-curves of nearly a hundred stars. The observations have been made in many ways: with the sliding-prism polarizing photometer, with the meridian, selenium, Zöllner, and wedge photometers, by measures of extra-focal plates, by estimates on the Harvard photographs, and by Argelander's method of visual estimates. The new methods of obtaining orbits from light-curves have so greatly diminished the labor of computation that it has been possible to develop in a relatively short time this branch of double-star astronomy. The catalogue of orbits given below contains 87 systems—a number that compares favorably with the lists of orbits of spectroscopic binaries and of visual double stars. Two or more solutions were made for each system. Of the total of 199 orbits, two were computed by Dugan, two by Stebbins, three by Roberts, eight by Russell, and 184 by the writer. The reader is referred to papers published during the last year for the theory of the orbits of eclipsing stars, and for examples of the solution for well-observed stars with many different types of light-curves.[1] The detailed discussion of the observational and computational

[1] H. N. Russell, "On the Determination of the Orbital Elements of Eclipsing Variable Stars," *Astrophysical Journal*, **35**, 315, 1911, and **36**, 54, 1912; "Elements of the Variables *W Delphini*, *W Ursae Majoris*, and *W Crucis*," *ibid.*, **36**, 133, 1912; H. N. Russell and H. Shapley, "On Darkening at the Limb in Eclipsing Variables," *ibid.*, **36**, 239 and 385, 1912; H. Shapley, "Elements of the Eclipsing Variables *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*," *ibid.*, **36**, 269, 1912; "The Visual and Photographic Ranges and Provisional Orbits of *Y Piscium* and *RR Draconis*," *ibid.*, **37**, 155, 1913; "The Orbits of *RZ Ophiuchi* and *ε Aurigae* Treated as Eclipsing Binaries," *Astronomische Nachrichten*, **194**, 225 (1913).

work, together with the results of the statistical investigation of the orbits, is to appear as a publication of the Princeton University Observatory.

In a work of this kind the investigation of every star cannot, of course, be considered as exhaustive and definitive. I have not utilized all the existing observations of the variables considered, nor tried to harmonize, explain, and adjust non-homogeneous sets of measures. The computations for each star have been based on what appeared to be the most complete and reliable series of observations, generally the work of some one observer being used, but occasionally the combined results of two or more; and in many cases the work in whole or in part has been based on unpublished photometric observations of my own. Whenever a good series of photometric measures has been available, estimates made by the Argelander method have been rejected. I am under obligation to a number of astronomers for assisting me with this study in various ways, but particularly to Professor Russell, who has directed and encouraged the investigation throughout and has helped with the computations in many cases; to Professor Pickering and Miss Cannon, who generously put at our disposal extensive unpublished photometric data and made special investigations of the spectra of many stars; and to Professor Nijland, of Utrecht, who has sent in manuscript light-curves based on long series of observations of 35 eclipsing systems, nearly one-half of which were stars for which no other data would have been available.

The stars in the following table have been divided into three general classes. In each division I have attempted to arrange the individual systems in order of the completeness of the photometric data, rather than in order of the degree of determinateness of the orbit obtained. The classification and order can be only approximate; but in general, stars in the first group have been so well observed that further photometric work will not appreciably change the solutions; orbits in the second group are susceptible of more or less improvement, as they are based on observations that are not as complete or as accurate as might be desired; while in the third class are listed those stars for which the observational data are very meager and uncertain, but concerning whose light-

variations enough is known to make it possible to derive approximate orbits. Further observations will probably alter greatly some of the orbits in the third group. But certain factors in these systems (for instance, the most important one of all—the density) are derived with a precision sufficient to aid materially in the generic studies of eclipsing variables. The manuscript sent by Professor Nijland contains only the co-ordinates of smooth curves drawn to represent his series of observations at primary eclipse. The precision of the resulting orbits cannot be estimated without a knowledge of the accuracy with which the normal points are represented and of the uniformity of the distribution of the observations. Consequently I have placed all the orbits that depend only on the Nijland curves in a group by themselves, arranging them in order of number of observations involved. Doubtless some of the curves are of high accuracy, while others must be considered only provisional.

EXPLANATION OF THE TABLE OF ORBITS

Letters in column (3), indicating the observer, have the following significance (the complete bibliography will be given in the later publication):

B = Baker	L = Miss Leavitt	Ro = A. Roberts
C = Miss Cannon	Le = Lehnert	Se = Seares
D = Dugan	Lu = Ludendorff	Sh = Shapley
E = Enebo	Lz = Luizet	St = Stebbins
G = Graff	N = Nijland	Sw = Stratonow
H = Haynes	P = E. C. Pickering	W = Wendell
I = Ichinohe	Pa = J. A. Parkhurst	Wh = Miss Whiteside
In = Innes	Pr = Pračka	Wy = Wylie
J = Jordan	*r* = see remark	

Unpublished observations are indicated by "ms" in this column. In column (4) the period of revolution is rounded off to the third decimal place.[1] The magnitude at maximum is only approximate for most faint stars; its precise value is not important. The ranges, column (6), are "unrectified," that is, the variations due to

[1] For corrections to the light-elements of many stars, obtained during the course of the work, see *Popular Astronomy*, December 1912; March 1913; and *Astronomische Nachrichten*, **192**, 79, 1912. Also see note below on *RS Cephei*.

eclipse, ellipticity, and "reflection" are combined. The first number for each system pertains to primary minimum, and the second number to secondary minimum. When the secondary is computed, but has not been observed, the value is inclosed in brackets; when it is assumed for purposes of solution, it is in parentheses, and when observed, no brackets or parentheses are used. The computed secondary minima are always for "uniform" disks, the "darkened" values being about twice as great except where central transit restrictions exist. Spectra are taken from *H.A.*, **56**, VI, with many revisions and additions furnished by Miss Cannon. "*tf*" signifies "too faint to classify."

In the absence of definite knowledge concerning the degree of darkening toward the limb of the stellar disks, I have computed double sets of elements for all systems on the two extreme hypotheses of uniform disks and disks completely darkened at the edge. These solutions are designated by "U" and "D" in the eighth column. For some stars, for which the orbit is indeterminate between certain limits, I have given the limiting solutions, "uniform" or "darkened"; and for some (*RZ Cassiopeiae* and *U Coronae*, for instance), solutions depending on different assumptions concerning the secondary minimum. The units of light, length, and density are respectively the maximum light of the system, the radius of the relative orbit, and the solar density. L_b is the light of the brighter star; that of the fainter is $L_f = 1 - L_b$. Columns (10) and (11) contain r_b and r_f, the radii of the two components. When the stars are elliptical these columns contain their longest axes, a; the shorter equatorial axes, b, may be obtained from column (13), which contains b/a. Column (12) contains the cosine of the orbital inclination, that is, the projected distance of centers at the time of mid-eclipse. When $\cos i$ is given as (0) the assumption of a central eclipse was necessary—the elements that would naturally develop from the observations yielding an imaginary value of i. With the third-class stars, however, $\cos i = (0)$ often means that in the absence of a good light-curve the simpler solution of central transit was found to represent the observations satisfactorily. Columns (14) and (15) contain the densities of the brighter and fainter stars, computed in all cases on the assumption

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES

GRADE I. STARS WELL OBSERVED

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	*Z Draconis*	D	1^d357	10^m46	2^m55	*lf*	U_1	0.911	0.217	0.270	0.074	0.986	0.36	0.19	15.8
					0.06		U_2	.886	.214	.270	.070	.990	.39	.19	12.3
							D_2	.927	.257	.262	.054	.997	.22	.21	13.1
2	*RT Persei*	D	0.849	10.62	1.37	F?	U_1	.861	.274	.274	.076	.980	.45	.45	6.2
					0.17		U_2	.849	.272	.272	.078	.980	.48	.48	5.62
							D_2	.879	.320	.256	.026	.987	.29	.57	4.64
3	*β Aurigae*	St	3.960	2.07	0.09	Ap	U	.50	.146	.146	.220	.990	.14	.14	1.0
					0.09		D	.50	.159	.159	.229	.993	.11	.11	1.0
4	*β Persei*	St	2.867	2.2	1.22	B8	U_1	.898	.210	.239	.134	....	.088	.060	11.4
					0.06		U_2	.907	.208	.228	.121	....	.092	.070	11.6
							D_2	.926	.241	.229	.129	....	.060	.069	11.3
5	*RZ Centauri*	L	1.876	8.48	0.46	A	U_1	.74	.490	.245	.221	.896	.020	.16	0.7
					0.34		D_1	.74	.481	.226	.239	.935	.020	.19	0.6
							D_2	.82	.491	.233	.000	.940	.018	.17	1.0
6	*U Pegasi*	W	0.375	9.32	0.60	F?	U_1	.57	.50	.50	.264	.887	.....	.....	.....
					0.46		U_2	.603	.450	.450	.300	.770	.88	0.88	1.52
							U_3	.788	.544	.348	.223	.782	.49	1.86	1.52
							D_2	.603	.454	.454	.352	.858	.70	0.70	1.52
7	*WZ Cygni*	Sh	0.584	9.9	1.45	A	U	.794	.455	.387	.058	.842	.30	.49	2.8
		ms			0.44		D	.794	.473	.369	.094	0.904	.23	.48	2.4
8	*S Cancri*	W	9.485	7.98	2.12	A	U	.858	.075	.203	.119	....	.178	.009	44.0
					0.04:		D	.858	.096	.184	.080	....	.084	.012	22.0
9	*SW Cygni*	W	4.573	9.06	2.66	A	U	.914	.131	.266	.110	....	.141	.017	43.0
					[0.02]		D	.914	.166	.247	.044	....	.070	.021	24.0
10	*U Cephei*	W	2.493	6.78	2.39	A	U	.890	.205	.324	.000	....	.126	.032	20.0
					0.05:		D	.890	.219	.319	(0)	....	.104	.034	17.0
11	*W Delphini*	W	4.806	9.40	2.70	A	U	.917	.135	.256	.114	....	.118	.017	40.0
					[0.03]		D	0.917	0.170	0.241	0.068	....	0.060	0.021	22.0

12	*RZ Cassiopeiae*	PaJ	1.195	6.43	1.22	A	U_1	1.00	0.273	0.257	0.118	....	0.24	0.28	∞
					(0.00)		U_2	0.913	.261	.269	.118	....	.27	.24	11.0
					or		D_1	1.00	.320	.256	.111	....	.14	.28	∞
					(0.07)		D_2	0.936	.313	.263	.106	....	.15	.26	10.0
13	*RX Herculis*	Sh	1.779	7.0	0.49	B9	U_1	.50	.190	.190	.084	....	.31	.31	1.0
		ms			0.49		U_2	.639	.213	.162	.051	....	.22	.50	1.0
							D_1	.50	.202	.202	.110	....	.26	.26	1.0
14	*V Serpentis*	L	3.454	9.52	0.94	A	U	.587	.185	.296	.131	.947	.099	.024	3.6
					0.24		D	.587	.224	.280	.098	.968	.053	.027	2.2
15	*U Sagittae*	W	3.381	6.43	2.76	B8	U	.921	.220	.291	.000	....	.055	.024	20.0
					[0.05]		D	.917	.238	.288	(0)	....	.043	.025	17.0
16	*RX Draconis*	HSh	3.786	10.20	0.50	F	U_1	.50	.099	.099	.037	....	.49	.49	1.0
		ms			0.50		U_2	.63	.111	.086	.024	....	.35	.75	1.0
							D_1	.50	.104	.104	.170	....	.42	.42	1.0
17	*u Herculis*	W	2.051	4.61	0.71	B3	U	.715	.312	.372	.249	.887	.067	.039	3.6
					0.24		D	.715	.308	.362	.278	.931	.064	.039	3.5
18	*U Ophiuchi*	PW	1.677	5.67	0.69	B8	U_1	.637	.224	.276	.048	.910	.14	.26	1.15
					0.59		U_2	.535	.252	.252	.079	.909	.18	.18	1.15
							D_2	.535	.252	.252	.109	.945	.17	.17	1.15
19	*ST Carinae*	L	0.902	9.31	0.87	A	U	.865	.273	.345	.245	....	.40	.20	10.3
					0.24		D	.868	.291	.363	.276	....	.33	.17	10.3
20	*RW Tauri*	Sh	2.769	8.05	3.42	B5	U_1	.957	.135	.254	.110	....	.36	.054	79.0
		ms			[0.02]		D_1	.957	.169	.241	.060	....	.18	.063	45.0
							U_2	.950	.136	.234	.084	....	.35	.069	56.0
21	*ZZ Cygni*	Sh	0.629	10.59	1.09	A?	U	.893	.287	.354	.215	....	.71	.38	12.7
		ms			0.06:		D_1	.918	.332	.357	.226	....	.46	.38	13.0
							D_2	.933	.361	.340	.197	....	.36	.43	12.3
22	*UW Cygni*	W	3.451	10.55	2.57	A?	U	.907	.175	.208	.000	....	.099	.063	13.0
					[0.08]		D	.907	.196	.209	.000	....	.075	.062	11.0
23	*R Canis Majoris*	W	1.136	5.38	0.60	F	U_1	.871	.346	.277	.220	....	.13	.25	4.3
					0.09		D_1	.782	.336	.336	.308	....	.14	.14	3.6
							U_2	.904	.266	.213	.140	....	.28	.55	6.0
							U_3	0.869	0.288	0.257	0.196	....	0.22	0.31	5.3

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
24	*W Ursae Majoris*....	r	0.334	7.91	0.60	G	U_1	0.760	0.431	0.242	0.143	0.757	1.32	7.39	1.0
					0.60		U_2	.500	.366	.366	.234	.745	2.22	2.22	1.0
							D_2	.500	.368	.368	.260	.850	1.72	1.72	1.0
25	*W Crucis*..........	L	198.5	8.90	0.60	Gp	U	.676	.520	.197	.150	.855	2×10^{-6}	3×10^{-5}	0.3
					0.28		D	.904	.613	.343	.247	.912	10^{-6}	5×10^{-6}	3.0
26	*RR Centauri*........	Ro	0.606	7.38	0.44	F	U	.563	.506	.506	.535	.631	0.35	0.35	1.3
					0.42		D	.574	.500	.500	.668	0.685	.31	.31	1.4
27	*RS Sagittarii*.......	Ro	2.416	6.10	0.74	A	U	.824	.324	.253	.076	....	.034	.071	2.85
					0.21		D	.813	.347	.277	.148	....	.028	.055	2.8
28	*S Velorum*..........	Ro	5.934	7.85	1.39	A	U	.722	.104	.230	.000	....	.170	.016	13.0
					[0.06]		D	0.722	0.121	0.233	(0)	....	0.106	0.015	9.6

GRADE II. STARS FAIRLY WELL OBSERVED

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
29	*RW Monocerotis*.....	HSh	1.906	8.75	1.72	A	U	0.795	0.207	0.287	(0)	....	0.21	0.078	7.5
		ms			0.10:		D	.795	.218	.282	(0)	....	.18	.082	6.5
30	*V Puppis*...........	Ro	1.454	4.14	0.64	B1	U	.60	.456	.420	0.236	0.813	.051	.065	1.27
					0.53	B3	D	.60	.461	.420	.289	.885	.042	.055	1.24
31	*Y Piscium*..........	C	3.766	9.00	3.40	A	U	.975	.144	.240	.107	....	.157	.034	84.0
		ms			(0.01)		D	0.975	.178	.231	.075	....	.084	.038	66.0
32	*U Coronae*..........	W	3.452	7.52	1.18	A	U_1	1.00	.188	.265	.193	....	.084	.030	∞
					(0.00)		U_2	0.95	.183	.269	.189	....	.093	.029	37.0
					(0.02)		U_3	.86	.176	.274	.180	....	.103	.028	15.0
					(0.05)		U_4	.74	.167	.273	.148	....	.120	.028	8.0
					(0.10)		D_3	.83	.202	.273	.168	....	.069	.028	9.0
33	*SZ Centauri*........	L	4.108	8.18	0.65	A	U_1	.59	.302	.242	.046	.933	.017	.032	0.9
					0.58		U_2	.626	.302	.242	.046	.933	.017	.032	1.1
							D	0.59	0.296	0.219	0.074	0.959	0.017	0.041	0.8

34	*Z Herculis*	W	3.993	7.10	0.80	F	U	0.508	0.097	0.207	0.110	....	0.45	0.047	4.7
					0.12		D	.570	.117	.217	.139	....	.26	.040	4.5
35	*U Scuti*	W	0.955	9.67	0.96	A	U	.900	.393	.295	.077	0.841	.17	.40	5.1
					0.30		D	.900	.418	.284	.121	.904	.12	.39	4.2
36	*Z Vulpeculae*	B	2.455	7.80	1.65	A	U	.749	.251	.314	.055	.872	.094	.048	4.7
					0.34		D	.749	.281	.296	.014	.922	.060	.051	3.3
37	*δ Librae*	W	2.327	4.83	1.10	A	U	.945	.324	.278	.104	....	.037	.058	13.0
					0.05:		D	.95	.386	.290	.098	....	.022	.051	11.0
38	*β Lyrae*	Sw	12.916	3.36	0.97	B8}p	D	.60	.271	.678	.499	.758	.0035	.0002	9.4
					0.45	B5}									
39	*RT Lacertae*	LzE	5.074	9.06	1.06	G5?	U	.602	.343	.343	.000	.949	.007	.007	1.5
					0.61		D	.602	.295	.295	.000	0.968	.011	.011	1.5
40	*SU Centauri*	L	5.354	8.73	0.87	F2	U	.926	.225	.203	.115	....	.021	.028	10.2
					0.06:		D	.934	.253	.213	.136	....	.014	.024	10.0
41	*RR Draconis*	Sh	2.831	9.98	2.96	A	U	.934	.099	.249	.136	....	.86	.054	90.0
		ms			[0.01]		D	.934	.131	.226	.076	....	.37	.072	42.0
42	*UZ Cygni*	W	31.304	10.29	1.88	A	U	.823	.070	.174	.049	....	.020	.0013	29.0
					[0.03]		D	.823	.084	.167	.000	....	.012	.0011	18.5
43	*VW Cygni*	G	8.431	10.32	1.94	A	U	.832	.106	.222	.036	....	.078	.009	22.0
					[0.04]		D	.832	.117	.217	(0)	....	0.059	0.009	17.0
44	*ε Aurigae*	Lu	9905.	3.26	0.75	F8p	U_1	.50	.030	.298	.249	....	3×10^{-6}	3×10^{-9}	100.0
							U_2	.50	.059	.170	(0)	....	3×10^{-7}	10^{-8}	8.0
					r		D_1	.50	.031	.307	.225	....	2×10^{-6}	2×10^{-9}	100.0
							D_2	.50	.069	.173	(0)	....	2×10^{-7}	10^{-8}	6.0
45	*RW Capricorni*	Sh	3.392	9.2	1.45	A	U	.737	.206	.237	.031	....	0.067	0.044	3.7
		ms			0.24		D	.788	.229	.229	.045	....	.049	.049	3.7
46	*RT Scuti*	Wh	0.512	9.65	0.71	F	U	.941	.435	.524	.442	0.800	.49	.28	23.0
					0.20:		D	.955	.498	.498	.452	0.880	.27	.27	21.0
47	*RS Cephei*	W	12.42	10.19	1.66	Ap	U	.783	.057	.229	.133	....	.23	.004	58.0
		ms			[0.02]		D	.783	.079	.197	.026	....	.089	.006	22.0
48	*SV Centauri*	L	1.661	8.80	0.86	A	U	.72	.323	.323	.122	....	.072	.072	2.6
					0.26		D	.771	.369	.314	.142	....	.049	0.079	4.7
49	*RZ Ophiuchi*	SeN	261.8	9.75	0.83	G8	U	.53	.037	.185	.112	....	.002	2×10^{-5}	28.0
		ms			[0.02]	r	D	0.53	0.046	0.154	0.047	....	0.001	3×10^{-5}	12.0

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
50	*SS Centauri*	L	2.479	8.73	1.55	B9	U	0.939	0.174	0.272	0.167	. . . .	0.21	0.055	30.0
					(<0.05)		D	1.00	.213	.263	.146	. . . .	.11	.060	∞
51	*SS Carinae*	L	6.602	12.29	0.66	*tf*	U_1	0.544	.113	.123	.010	. . . .	.106	.083	1.0
					0.66		U_2	.500	.118	.118	.016	. . . .	.094	.094	1.0
							D_2	.500	.115	.115	.016	. . . .	.10	.10	1.0
52	*WW Cygni*	G	3.318	10.00	2.91	Ap	U	.931	.188	.265	.073	. . . .	.091	.032	27.0
					[0.04]		D	.931	.226	.258	.016	. . . .	.053	.035	14.0
53	*RR Velorum*	Ro	1.854	10.00	0.91	A	U	.691	.140	.197	.108	. . . .	.71	.25	4.4
					0.15		D	0.767	0.166	0.195	0.120	. . . .	0.42	0.26	4.5

STARS WITH LIGHT-CURVES DETERMINED BY NIJLAND

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
54*	*RY Persei*	N	6.864	8.20	2.37	A2	U	0.887	0.144	0.252	0.081	. . . .	0.048	0.009	24.0
		ms			[0.04]		D	.887	.179	.242	.000	. . . .	.025	.010	14.0
55	*RW Geminorum*	N	2.865	9.80	2.14	A	U	.861	.217	.319	.004	. . . .	.080	.025	14.0
		ms			[0.07]		D	.861	.248	.310	(0)	. . . .	.054	.027	10.0
56	*Z Persei*	N	3.056	9.60	2.70	A	U	.917	.116	.232	(0)	. . . .	.46	.057	44.0
		ms			[0.02]		D	0.917	.132	.231	(0)	. . . .	.31	.058	34.0
57	*Y Camelopardalis*	N	3.306	9.75	2.02		U	1.00	.180	.243	.121	. . . .	.106	.043	. . . .
		ms	.		(0.00)		U	0.844	.171	.244	.073	. . . .	.123	.043	11.0
					(0.09)		D	0.844	.207	.238	.030	. . . .	.069	.045	7.0
58*	*RR Delphini*	N	4.599	10.50	1.20	A	U_1	1.00	.092	.341	.313	. . . .	.40	.008	∞
		ms			(0.00)		U_2	0.669	.106	.234	.128	. . . .	.27	.025	9.6
					(0.08)		D_2	.673	.136	.216	.087	. . . .	.13	.032	5.2
59	*ST Persei*	N	2.648	9.80	2.08	A	U	.853	.169	.264	.083	. . . .	.20	.052	14.0
		ms			[0.07]		D	.853	.206	.253	.000	. . . .	.11	.058	8.8
60	*RV Persei*	N	1.974	10.75	2.20	A	U_1	.954	.291	.434	.208	. . . .	.070	.021	45.0
		ms			(0.02)		U_2	.868	.290	.427	.137	. . . .	.070	.022	14.0
					(0.07)		D_2	0.870	0.355	0.418	0.065	. . . .	0.039	0.024	9.0

								L_b	r_b	r_f		b/a	e_b	e_f	J_b/J_f
61	*SY Andromedae*	N	34.912	10.65	1.50	A?	U	0.749	0.036	0.119	(o)	....	0.120	0.0033	33.0
		ms			[0.02]		D	.749	.042	.120	(o)	....	0.075	.0032	24.0
62	*RW Ursae Majoris*	N	7.328	10.35	1.05	G?	U	.62	.025	.249	0.186	....	1.00	.008	41.0
		ms			[0.01]		D	.62	.071	.202	.112	....	0.35	.015	13.0
63	*TW Draconis*	N	2.806	7.30	1.60	B9	U	.771	.130	.371	.244	....	.39	.017	26.0
		ms			(0.03)		D	0.780	.180	.334	.177	....	.14	.023	12.0
64	*TT Lyrae*	N	5.244	9.45	2.20	A	U_1	1.00	.128	.284	.198	....	.118	.011	∞
		ms			(0.00)		U_2	0.868	.132	.254	.122	....	.106	.015	25.0
					(0.04)		D_2	.877	.152	.253	.116	....	.070	.015	20.0
65	*TT Andromedae*	N	2.765	11.30	1.30	A	U	.698	.158	.244	.084	....	.22	.060	5.5
		ms			[0.15]		D	.803	.177	.253	.141	....	.16	.055	8.4
66	*SY Cygni*	N	6.006	10.90	2.30	G5?	U	.88	.090	.237	(o)	....	.25	.014	51.0
		ms			[0.02]		D	.88	.100	.237	(o)	....	.19	.014	42.0
67	*RS Vulpeculae*	N	4.477	7.35	0.68	A	U	.535	.272	.082	.030	....	.017	.61	0.10
		ms			[0.05]		D	.535	.252	.088	(o)	....	.021	.50	0.14
68	*TV Cassiopeiae*	N	1.813	7.35	1.00	B9	U	.925	.248	.261	.155	....	.14	.12	14.0
		ms			(0.05)		D	.923	.272	.272	.176	....	.10	.10	12.0
69	*3.1911 (Cancri)*	N	10.174	10.10	1.55	tf	U	.76	.048	.194	.105	....	.57	.009	51.0
		ms			[0.02]		D	.76	.057	.190	.096	....	.35	.010	35.0
70	*VV Cygni*	N	1.477	12.85	0.75	tf	U	.832	.211	.222	.149	....	.33	.28	5.5
		ms			(0.10)		D	.874	.251	.224	.161	....	.20	.27	5.5
71	*RV Lyrae*	N	3.599	11.60	1.90	A	U	.826	.089	.296	.191	....	.73	.020	53.0
		ms			[0.02]		D	0.826	0.126	0.258	0.107	....	0.26	0.031	20.0

GRADE III. STARS INSUFFICIENTLY OBSERVED

72	*RZ Draconis*	W	0.551	9.97	0.80	Ap	U	0.90	0.34	0.44	0.330	0.815	0.82	0.39	15.0
					0.22		D	.96	.40	.40	.332	.886	.43	.43	22.0
73	*SZ Herculis*	Sh	0.818	9.5	1.48	tf	U	.74	.25	.33	.019	....	.62	.28	4.9
		ms			[0.18]		D	.74	.27	.32	.000	....	.54	.31	4.2
74	*SV Tauri*	E	2.167	9.37	0.72	A	U	.96	.27	.19	.000	....	.075	.21	12.0
					0.05:		D	0.95	0.30	0.19	(o)	....	0.051	0.20	7.5

CATALOGUE OF THE ORBITS OF ECLIPSING BINARIES—*Continued*

No.	Star	Obs.	Period	Max.	Range	Sp.	Sol.	L_b	r_b	r_f	cos i	b/a	ρ_b	ρ_f	J_b/J_f
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
75	*RX Cassiopeiae*	W	32.315	8.66	0.69	KO	U	0.57	0.27	0.27	0.136	0.809	0.0005	0.0005	1.3
					0.55		D	.57	.28	.28	.170	.880	.0004	.0004	1.3
76	*RZ Scuti*	BWy	15.194	7.47	1.38	B3	U	.88	.143	.298	.216	....	.010	.0011	29.0
					0.03:		D	.90	.157	.314	.238	....	.008	.0010	36.0
77	*Y Cygni*	W	2.996	6.95	0.60	A	U	.58	.167	.167	.070	....	.16	.16	1.4
		ms			0.40		D	.58	.166	.166	.088	....	.16	.16	1.4
78	*SX Cassiopeiae*	Lz	36.572	8.68	1.00	G3	U	.53	.21	.32	.115	.846	.0008	.0002	2.7
					0.41		D	.66	.26	.32	.180	.905	.0003	.0002	2.9
79	*SY Centauri*	L	6.631	9.88	0.87	A	U	.76	.15	.45	.39	....	.042	.002	28.0
					(0.02)		D	.79	.18	.45	.39	....	.026	.002	33.0
80	*SW Centauri*	L	5.219	9.12	2.33	A	U	.88	.09	.23	(0)	....	.32	.021	47.0
					0.02		D	.88	.10	.23	(0)	....	.22	.021	36.0
81	*RS Scuti*	I	1.329	8.86	0.93	F	U	.50	.26	.26	.00	.848	.30	.30	1.0
					0.93		D	.50	.27	.27	.00	.905	.24	.24	1.0
82	*RZ Aurigae*	Pr	3.011	10.5	1.74	*tf*	U	.80	.21	.27	.05	....	.080	.036	6.7
					[0.13]		D	.80	.26	.27	.00	....	.044	.036	4.2
83	*RW Persei*	P	13.199	9.5	2.2	A	U	.87	.06	.16	(0)	....	.14	.009	41.0
					[0.02]		D	.87	.08	.16	(0)	....	.09	.009	30.0
84	*RR Puppis*	In	6.430	9.45	1.11	A	U	.64	.10	.24	(0)	....	.17	.012	10.0
					[0.07]		D	.64	.11	.24	(0)	....	.12	.013	8.0
85	*Y Leonis*	Le	1.686	9.4	2.75	A	U	.92	.16	.26	.10	....	.60	.13	31.0
					[0.03]		D	.92	.20	.25	.07	....	.31	.14	19.0
86	*SX Sagittarii*	P	2.077	8.58	0.82	A5	U	.53	.19	.44	(0)	....	.22	.018	6.1
					[0.10]		D	.53	.25	.40	(0)	....	.11	.23	2.3
87	*X Carinae*	Ro	1.083	7.9	0.8	A	U	0.52	0.50	0.50	0.10	0.878	0.06	0.06	1.1

that the mass of each system is equally divided between the two components. This assumption will in general give the density for the bright star too low, and for the faint companion too high. The densities are usually given to the second significant figure, though they are often entirely uncertain in the last place. In the final column is given the ratio of the surface intensity of the star that has a majority of the light, to the surface intensity of the other. This ratio is greater than unity except in rare instances where the star that has the most light has a lower intensity per unit area.

AUXILIARY TABLES

1. Eccentricity of orbit has been determined for the stars listed below. In some other cases the orbits are known to be practically circular, but in most systems the evidence is insufficient. When only $e \cos \omega$ has been found from the displacement of the secondary minimum, I have assumed for this table $\omega = (0°)$ or $(180°)$, which gives minimum eccentricity; the uncertainty of such a determination justifies giving only circular elements in the catalogue. Spectrographic data were available for Nos. 17, 23, and 37.

	Star	Eccent.	Long. of Periastron		Star	Eccent.	Long. of Periastron
1	*Z Drac.*	0.010	(0°)	25	*W Cruc.*	0.06	(180°)
2	*RT Pers.*	.012	(0)	27	*RS Sag.*	.092	261.1
17	*u Herc.*	.053	66.9	37	*δ Libr.*	.054	29.2
19	*ST Car.*	.052	(0)	78	*SX Cass.*	.043	(180)
23	*R Can. Maj.*	0.138	90	87	*X Carin.*	0.02	165

2. With the aid of spectrographic data it is possible to find for four systems the radius of the orbit, dimensions of the stars, and

Star			Masses		Maxima Radii		Densities		Distance of Centers
			m_b	m_f	r_b	r_f	ρ_b	ρ_f	
3	*β Aurig.*	Unif.	2.38	2.34	2.58	2.58	0.14	0.14	17.7
		Dark	2.40	2.36	2.81	2.81	0.11	0.11	17.7
17	*u Herc.*	Unif.	7.50	2.87	4.60	5.48	0.097	0.022	14.7
		Dark	7.66	2.93	4.56	5.35	0.095	0.022	14.8
30	*V Pupp.*	Unif.	18.7	18.7	8.23	7.57	0.051	0.065	12.5
		Dark	19.4	19.4	8.45	7.70	0.042	0.055	12.7
38	*β Lyrae*	Dark	1.42	14.2	16.2	40.6	0.0006	0.0004	59.9

actual masses and densities—all in terms of the sun. The masses in the system of *V Puppis* are assumed equal. The mass ratio for *β Lyrae* was taken as 10/1 (see note).

3. A "reflection" effect has been detected in a few accurately observed stars, and no doubt its occurrence would be found quite general if the precision of the observations was increased. The bright side of the companion (toward the primary) gives out the light $1-L_b$ (see ninth column of the catalogue); the light of the opposite side is less by 0.040 for *Z Draconis*, 0.022 for *RT Persei*, 0.044 for *β Persei*, 0.025 for *RZ Centauri*, and 0.03 for *Z Herculis*.

NOTES TO THE CATALOGUE AND TABLES

1. *Z Draconis.*—The accuracy of the light-curves of the first four stars greatly exceeds that of all other eclipsing binaries. Their orbits will be discussed *in extenso* in a paper soon to be published. The solutions U_1 for *RT Persei* and for *Z Draconis* are by Dugan; other solutions for *Z Draconis* and D for *β Aurigae* are by Russell; U for *β Aurigae* and U_1 for *Algol* are by Stebbins.

6. *U Pegasi.*—First solution by Roberts, assuming stars in contact, *M.N.*, **66**, 135, 1906. Solutions indeterminate over a small range.

8. *S Cancri, SW Cygni, U Cephei, W Delphini.* See *Astrophysical Journal*, **36**, 269, 1912.

13. *RX Herculis*, 15. *U Sagittae.*—Harvard classifies spectra as A; Frost mentions helium lines; see *Astrophysical Journal*, **22**, 214, 215, 1905.

23. *R Canis Majoris.*—These orbits are based on Wendell's observations of 1898–1899 which show the secondary minimum exactly halfway between successive primaries. Jordan's elements from Allegheny spectrograms give $e=0.138$, $\omega=196°$ in 1908. Hence the line of apsides must be in motion, but there are no data to estimate its rate of revolution. With this value of the eccentricity, and considering that primary eclipse occurred at periastron, solutions U_1 and D_1 are obtained. They represent the observations satisfactorily. For the purpose of illustration U_2, which assumes the primary at apastron and fails to fit the secondary minimum well, and U_3, which is the set of circular elements, are given.

24. *W Ursae Majoris.*—Observations by Müller, Kempf, and Baldwin; solutions by Russell; see *Astrophysical Journal*, **36**, 139, 1912.

25. *W Crucis.*—Uniform solution by Russell; see *Astrophysical Journal*, **36**, 146, 1912.

26. *RR Centauri.*—Uniform solution by Roberts, *M.N.*, **63**, 545, 1904.

31. *Y Piscium*, 41. *RR Draconis.*—See *Astrophysical Journal*, **37**, 155, 1913.

38. *β Lyrae.*—The orbits previously obtained by Stein, Myers, Roberts, and von Hepperger are not consistent with the statement of Curtiss in *Alleg. Bull.*, **2**, 115, 1911, that the star eclipsed at primary minimum has the stronger continuous spectrum. In a recent letter Curtiss estimates that the primary star of type B8 has 60 per cent of the light of the system. The primary eclipse must then necessarily be partial, with large faint star in front. I am able to find no possible "uniform" orbit, but the "darkened" solution represents the light-variations quite satisfactorily, and at the same time conforms to the first hypothetical system deduced by Curtiss from his extensive spectroscopic investigation. He finds that the brighter star has at most one-tenth the mass of the other.

39. *RT Lacertae.*—Uniform solution very unsatisfactory; probably definite evidence of darkening toward the limb. *Astrophysical Journal*, **36**, 401, 1912.

42. *UZ Cygni.*—I find that Wendell's photometric observations contradict Hartwig's visual estimates relative to the secondary minimum. *A.N.*, **165**, 121, 1904, *V.J.S.*, **39**, 254, 1904; **40**, 329, 1904.

44. *ε Aurigae*, 49. *RZ Ophiuchi.*—See recent discussion of orbits in *A.N.*, **194**, 225 (1913). Spectrum of *RZ Ophiuchi* is estimated G5 to K0.

47. *RS Cephei.*—I have determined new light-elements from Wendell's manuscript observations: Min. = J.D. 2417140.469, G.M.T. + $12^{d}4204$ E.

54–71. *Nijland's stars.*—Variables for which the series of observations have been completed by Professor Nijland are marked with asterisks.

72. *RZ Draconis.*—A complete study of this star is being made at Princeton.

73. *SZ Herculis.*—Comparison star probably variable.

77. *Y Cygni.*—Only circular elements are possible from existing data; according to Dunér the orbit is highly eccentric.

78. *SX Cassiopeiae.*—Spectrum G0 to G5.

85. *Y Leonis.*—Elements from rough light-curve; observed magnitudes not available.

CONCLUSIONS

Among the general results obtained from the present investigation the following points may be briefly mentioned. The complete statistical discussion will be published later.

1. The better the observations of an eclipsing binary are, the more satisfactory is the theoretical representation of the light-variations. Irregularities in the shape of light-curves disappear with increasing photometric accuracy. Halts and inflections in them have no objective existence, and the only apparently real peculiarities are occasional slight asymmetries and brightening

toward periastron (*Astrophysical Journal*, **36**, 278, 1912; **36**, 146, 1912).

2. The existence of darkening toward the limb of stellar disks is indicated in a large number of systems by the slightly better agreement of the "darkened" solution with the observed data, and its existence is actually demonstrated in a few cases. The degree of darkening, however, is as yet quite indeterminate.

3. In all but one of 28 first-grade stars, three of 25 second grade, and one of 16 of the third grade, there is a positive indication that the fainter star is self-luminous, and in no case is it necessary to assume one component completely black. In about two-thirds of the systems the difference in brightness of the components does not exceed two magnitudes, and no observed difference is greater than four magnitudes.

4. Regarding the relative sizes of the two components of an eclipsing system the following table shows that the conspicuous preponderance of systems in which the fainter star is the larger is entirely a matter of selection; and suggests, further, that there exist great numbers of eclipsing stars of small range in which the faint companion is smaller.

Range		Faint Star Large	Bright Star Large	Stars Nearly Equal
Greater than one magnitude	Unif.	47	2	2
	Dark	44	4	3
Less than one magnitude	Unif.	12	10	13
	Dark	9	12	14
Total	Unif.	59	12	15
	Dark	53	16	17

5. Whenever the relative color-index of the components has been determined from the difference between the photographic and visual ranges at total eclipse, the large faint star has been found to be the redder. These stars are therefore presumably of "later" spectral type than their primaries, but columns (14) and (15) of the catalogue show that they are almost certainly less dense. See *Astrophysical Journal*, **37**, 155 ff., 1913, for more complete discussion.

6. The relation between the separation of the components of a close system and the gravitational elongation is shown in the following table where the ratio of the equatorial axes, b/a, for "uniform" and "darkened" solutions is compared with Darwin's theoretical value for homogeneous, incompressible fluid.[1] In forming the groups in order of separation of the stellar disks, I have excluded *X Carinae*, for which the observational data are not available.

Number Stars	Mean Separation $1-a_b-a_f$	Uniform b/a	Darkened b/a	Darwin b/a
5	0.501	0.971	0.983	0.944
5	.399	.900	.939	.902
5	.315	.847	.906	.857
4	.196	.809	.883	.772
4	0.106	0.700	0.788	0.692

7. In forming a table showing the distribution of densities relative to spectra, the "darkened" values have been used; the relative distribution would be altered but little if the "uniform" densities had been taken.

Density	B	A	F	G	K
>1.00	...	...	...	1	...
1.00 to 0.50	...	...	1	...	...
0.50 to 0.20	1	10	6	1	...
0.20 to 0.10	4	12	1	1	...
0.10 to 0.05	3	17	...	...	...
0.05 to 0.02	2	8	...	...	...
0.02 to 0.01	...	3	1	1	...
0.01 to 0.001	2	...	...	...	...
0.001 to 0.0001	...	...	...	2	1
<0.0001	...	...	1	1	...
Total	12	50	10	7	1

The first-type stars (spectra B and A) show a marked preference for an intermediate density, 75 per cent of them coming between the values 0.02 and 0.20, while out of the 18 second-type stars only two fall into that interval, and for one of them a small and permissible change of the elements would take it out of these limits.

[1] See *Astrophysical Journal*, **36**, 62, 1912.

The second-type stars fall apparently into two groups, of which one precedes and one follows the first-type stars in order of density. These two groups are obviously identical with the two classes of second-type stars of very greatly different luminosity discussed by Hertzsprung[1] and Russell,[2] and the facts collected here afford direct support of Russell's theory that the differences in brightness of the two groups are to be ascribed in the main to great differences in the mean density.[3]

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[1] *Zeit. für wiss. Phot.*, **3**, 429; **5**, 86, 1907.

[2] *Astrophysical Journal*, **36**, 153, 1912.

[3] *Science*, N.S., **34**, 523, 1911; *Proc. Am. Phil. Soc.*, **51**, 569, 1912.

ORBIT OF THE SPECTROSCOPIC BINARY π^5 *ORIONIS*

By OLIVER J. LEE

The variable radial velocity of π^5 *Orionis* ($\alpha = 4^h 49^m$; $\delta = +2°$ $17'$; mag. $= 3.9$) was discovered and announced by Frost and Adams.[1] The spectrum of this star is classified as B 3 in the *Harvard Revised Photometry*. The lines are often faint and always diffuse and difficult to measure. No evidence of the spectrum of the second component has been found. The present discussion is based upon measures of 64 plates, seven of which were taken in 1902–1903 with the three-prism dispersion of the Bruce spectrograph, the other 57 plates were taken with one prism.

The following lines were used in obtaining the velocities:

Element	Wave-Length	Element	Wave-Length	Element	Wave-Length
H	3970.213	*He*	4143.919	*Mg*	4481.400
He	4009.417	*C*	4267.301	*Si*	4552.636
He	4026.342	*H*	4340.634	*Si*	4567.897
H	4101.900	*He*	4388.100	*Si*	4574.791
Si	4116.400	*He*	4437.718	*He*	4713.252
He	4120.973	*He*	4471.646	*H*	4861.527

The lines at λ 4143.919, λ 4340.634, λ 4388.100, and λ 4471.646 are usually best and they have been given most weight.

For the first seven plates given in the Table of Observations the measures of five are due to Adams, while the means of measures by Frost and Adams are given for B 475 and B 488. These early observations fall well along the velocity-curve, considering that three-prism dispersion is much too great for a spectrum of this type. They have been included in deriving the normal points. All the remaining measures are mine.

Since our observations of this star cover more than ten years, a precise determination of the period was possible from Bruce plates alone, and the value 3.70045 days was adopted as definitive. With this period and the resulting observational velocity-curve a preliminary set of elements was derived by the graphical method

[1] *Astrophysical Journal*, **17**, 150, 1903.

TABLE OF OBSERVATIONS

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
			Days			km	km
		1902					
A 332...	A	Mar. 4.608	1.23	3	2	+ 1	+ 5.7
B 469...	A	Dec. 17.712	0.69	5	4	+58	+10.7
B 475...	A	31.689	3.56	6	1	+71	− 9.9
		1903					
B 480...	A	Jan. 1.743	0.92	4	2	+32	+ 7.4
A 384...	A	16.710	1.09	6	3	+ 7	− 0.4
B 488...	A	21.562	2.24	4	2	−34	−12.5
A 390...	F	22.688	3.35	4	2	+73	+ 0.9
		1907					
I B 1276..	F	Dec. 6.635	2.39	6	5	− 9	+ 1.8
1278..	Fox, L	6.772	2.53	6	4	− 2	− 3.9
1280..	L	6.836	2.59	8	4	+14	+ 6.8
1285..	L	11.582	3.64	7	3	+80	0.0
1293..	F, L	20.630	1.59	4	4	−30	− 1.6
1295..	B	20.824	1.78	6	4	−34	− 1.0
1296..	B	28.537	2.10	5	4	−28	+ 0.6
1298..	B	28.708	2.27	8	3	−33	−13.2
1300..	Fox	30.631	0.49	6	5	+58	− 5.4
1303..	Fox	30.800	0.66	7	4	+51	+ 0.9
		1908					
1312..	L	Jan. 7.616	1.08	7	3	+14	+ 5.7
1314..	L	7.686	1.15	3	2	−11	−19.3
1326..	B	14.639	0.71	6	5	+47	+ 1.8
1329..	B	14.744	0.81	7	3	+29	− 8.3
1351..	F, B	20.557	2.93	6	4	+26	−13.2
1355..	B	20.676	3.05	6	3	+46	− 4.0
1357..	L	21.522	0.18	11	4	+70	− 9.8
1360..	L	21.656	0.31	12	4	+72	− 1.0
1744..	...	Sept. 18.956	1.09	6	2	+16	+ 8.6
1897..	L	Dec. 7.717	3.14	5	3	+51	− 5.7
1904..	B	14.856	2.88	8	5	+36	+ 1.7
		1909					
1989..	F	Mar. 1.588	1.90	8	4	−36	− 2.6
2160..	B	Oct. 25.828	3.31	9	3	+53	−16.9
2169..	L	29.876	3.67	9	4	+73	− 9.5
2174..	B	Nov. 5.764	3.15	9	4	+52	− 6.1
2190..	L	22.665	1.55	8	3	−37	−10.3
2208..	B	Dec. 17.624	0.60	5	3	+54	− 1.0
2219..	F	22.697	1.97	9	3	−40	− 7.3
2223..	L	22.861	2.13	6	2	−37	−10.0
		1910					
2231..	L	Jan. 3.547	2.72	10	3	+20	+ 1.7
2237..	L, B	3.780	2.95	10	3	+42	0.0
2248..	B	14.549	2.61	9	3	+12	+ 4.0
2255..	L	14.822	2.88	10	4	+29	− 5.9
2261..	L	18.556	2.93	10	4	+26	−13.2
2276..	L	Feb. 7.524	0.69	6	3	+41	− 6.3
2280..	L	14.589	0.35	6	3	+73	+ 0.9
		1912					
3100..	B	Sept. 30.961	1.33	8	4	−16	+ 2.9
3104..	L	Oct. 4.849	1.51	11	4	−19	− 5.1
3105..	L	4.889	1.55	14	5	−19	− 7.6

TABLE OF OBSERVATIONS—*Continued*

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
		1912	Days			km	km
I B 3114..	L	14.900	0.47	14	4	+81	+15.7
3115..	L	14.956	0.53	12	5	+71	+10.7
3122..	M	25.793	0.26	8	2	+87	+10.6
3123..	M, L	25.949	0.32	5	2	+86	+11.9
3125..	L	25.901	0.37	9	4	+80	+ 9.0
3135..	M	Nov. 4.683	2.75	8	2	+15	− 5.9
3138..	L	4.847	2.91	11	4	+35	− 2.4
3142..	M	6.825	1.19	10	4	+10	+11.1
3144..	M	6.974	1.34	9	4	−10	+ 3.6
3148..	L	8.767	3.13	11	5	+64	+ 6.2
3149..	L, B	8.803	3.17	11	5	+65	+ 5.2
3151..	B	8.958	3.33	12	4	+82	+10.6
3155..	L	18.734	1.99	9	3	−26	+ 5.9
3158..	M	18.892	2.15	9	3	−16	+10.0
3163..	B	19.699	2.96	11	5	+34	− 8.9
3164..	B	19.792	3.05	11	4	+58	+ 7.9
3169..	B	22.712	2.27	12	4	−15	+ 4.5
3170..	B	22.765	2.32	11	4	−12	+ 2.2

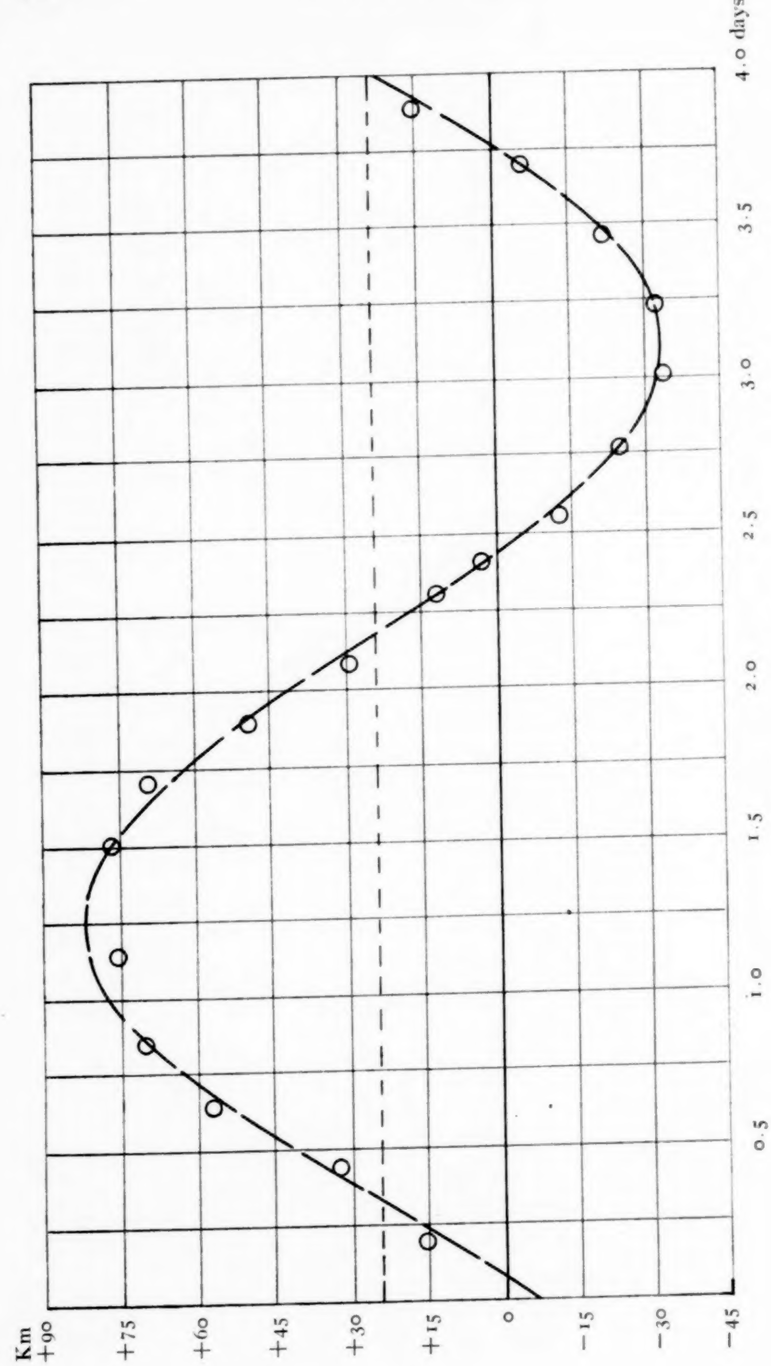
In column headed "Observer," A=Adams; B=Barrett; F=Frost; L=Lee; M=S. A. Mitchell. Mr. F. R. Sullivan assisted as usual in securing the plates.

TABLE OF NORMAL PLACES

No.	Phase	Limits of Phase	Velocity	Residual O.−C.	Wt.
	Days	d. to d.	km	km	
1..........	0.26	0.1 0.4	+76.5	0.0	0.6
2..........	0.46	0.4 0.6	+69.2	+3.9	0.5
3..........	0.64	0.6 0.7	+49.4	−1.7	0.6
4..........	0.83	0.7 0.9	+29.6	−3.9	0.2
5..........	1.06	1.1	+11.9	+0.8	0.3
6..........	1.16	1.1 1.2	+ 2.5	+0.8	0.3
7..........	1.30	1.3	−13.0	−2.8	0.3
8..........	1.52	1.5 1.6	−25.1	−0.3	0.5
9..........	1.76	1.8	−34.0	−1.0	0.1
10..........	1.98	1.9 2.1	−32.4	−0.1	0.5
11..........	2.21	2.1 2.3	−22.1	−1.1	0.6
12..........	2.45	2.4 2.5	− 5.9	+0.2	0.3
13..........	2.64	2.5 2.8	+15.2	+4.2	0.4
14..........	2.89	2.8 3.0	+32.4	−3.0	1.0
15..........	3.09	3.0 3.2	+57.3	+2.6	0.8
16..........	3.30	3.3	+70.3	+1.1	0.3
17..........	3.59	3.5 3.7	+75.4	−5.7	0.3

of Lehmann-Filhés.[1] This orbit was given at the Cleveland meeting of the American Association. Later it was concluded that this series of observations would justify the labor of making a least-

[1] *Astronomische Nachrichten*, **136**, 17, 1894.



Velocity-curve of π^5 *Orionis*

squares solution of the orbit. Accordingly the observations were grouped into 17 normal places depending upon the phase, and the weights were assigned. Equations of condition were formed with five unknowns, viz., $\delta\gamma$, δK, δe, $\delta\omega$, and δT. The differential formulae of Lehmann-Filhés (*loc. cit.*) were employed for the last four, while $\delta\gamma$ was added and was given a coefficient unity. In the resulting normal equations the similarity in form of the coefficients in the equations from $\delta\omega$ and δT showed that these quantities and therefore the others would not be well determined by this set of observations. The large value of the correction $\delta\omega$ and the reduction of the eccentricity from 0.051 to 0.010 by this solution showed that with observations so necessarily inexact as those of this star the work of determining an elliptical orbit having an eccentricity of the order of 0.01 would be useless. Hence a circular orbit was assumed having the following elements:

$$\begin{aligned} P &= 3.70045 \text{ days} \\ K &= 57.04 \text{ km} \\ \gamma &= +24.49 \text{ km} \\ T &= \text{J.D. } 2{,}417{,}921.64 \end{aligned}$$

T is an epoch of maximum positive velocity of the star. With these elements an ephemeris was computed and differential coefficients derived for the 17 normal places. For the reason given above no correction to the period was sought.

The resulting normal equations are:

$$\begin{array}{cccc} +7.600x & +0.222y & -0.586z & =-1.730 \\ & +3.196 & +0.072 & =+2.583 \\ & & +4.401 & =-1.865 \end{array}$$

$$x=\delta\gamma$$
$$y=\delta K$$
$$z=K\mu\delta T$$

These equations yield the following corrections:

$$\begin{aligned} \delta\gamma &= -0.289 \text{ km} \\ \delta K &= +0.839 \text{ km} \\ \delta T &= -0.005 \text{ day} \end{aligned}$$

The corrected elements are:

$$
\begin{aligned}
P &= 3.70045 \text{ days} && \pm 0.00002 \text{ day} \\
K &= 57.88 \text{ km} && \pm 1.07 \text{ km} \\
\gamma &= +24.20 \text{ km} && \pm 0.70 \text{ km} \\
T &= \text{J.D. } 2{,}417{,}921.64 && \pm 0.01 \text{ day} \\
a \sin i &= 2{,}945{,}000 \text{ km}
\end{aligned}
$$

The phases given for the plates and for the normal places are referred to the time of maximum velocity of recession. The residuals given in the Table of Observations are obtained by scaling from the final velocity-curve. The probable error of a single plate is ± 5.36 km. The residuals given in the Table of Normal Places are the differences between the observed value and the corrected ephemeris.

In only one instance do the residuals for the normal places derived from the final ephemeris differ more than 0.5 km from those obtained by a direct substitution in the equations of condition. Hence, in this case, it does not seem necessary to make a second solution.

In the preliminary investigation, made to find a satisfactory set of elliptical elements, the *Tables for the True Anomaly in Elliptic Orbits*, which form No. 17, Vol. 2, of the "Publications of the Allegheny Observatory," were found to be very convenient.

YERKES OBSERVATORY
March 1913

THE VARIATION OF THE SUN

By C. G. ABBOT, F. E. FOWLE, AND L. B. ALDRICH[1]

In the year 1902 preliminary experiments were begun at Washington to determine the solar constant of radiation. About 700 determinations of it have now been obtained, depending on observations at altitudes ranging from sea-level to 4420 meters. As originally devised by Langley, we determine spectral energy intensities and atmospheric transmission coefficients for numerous wave-lengths between about $0.30\,\mu$ in the ultra-violet and $2.5\,\mu$ in the infra-red, by spectrobolometric observations at high and low sun. The indications of the spectrobolometer are reduced to the standard scale of calories per square centimeter per minute by means of the readings of the pyrheliometer.

At the time when the observations were begun in 1902 there was no satisfactory establishment of the standard scale of pyrheliometry, nor indeed any pyrheliometer which was invariable relatively to itself from year to year. We at first made use of a modification of Tyndall's mercury pyrheliometer. This was improved in 1906 as the copper disk pyrheliometer, which has been in use on Mount Wilson ever since, and which is described in Vol. 2 of the *Annals of the Astrophysical Observatory*. A still later improvement took place in 1910 with the introduction of the so-called "Silver-Disk Pyrheliometer" which has attained considerable favor, and which is now in use in numerous countries. Neither of these instruments is capable of yielding independently the standard scale of radiation, but they possess the valuable qualities of simplicity and of being constant from year to year. Beginning with the year 1903 and extending until the end of the year 1912 we have repeatedly devised and experimented with instruments to fix the standard scale of radiation. Three of these instruments (called Water-Flow Pyrheliometers Nos. 2 and 3, and Water-Stir Pyrheliometer No. 4) have been tested with satisfactory results which are stated in a publica-

[1] Published by permission of the Secretary of the Smithsonian Institution.

tion by two of us.[1] We are now satisfied that the measurements made since 1903 can be reduced to the standard scale of radiation to within 1 per cent.

Measurements of the solar constant of radiation were begun at Washington, practically at sea-level, and were continued when favorable opportunities presented themselves from October 1902 until May 1907. Measurements were begun on Mount Wilson in California (elevation 1730 meters) in 1905, and have been continued with the exception of 1907 during about 6 months in the year in each of the succeeding years. Expeditions to Mount Whitney in California, altitude 4420 meters, were made in 1908, 1909, 1910. Expeditions to Bassour, Algeria, altitude 1160 meters, were conducted in the autumn of 1911 and the summer of 1912. In all 696 complete determinations of the solar constant of radiation have been made, and still others are unreduced. The differences found between the results at different elevations are very small, and seem attributable rather to experimental error or slight atmospheric irregularities than to any difference of elevation. The mean of all these 696 determinations made principally between the years 1905 and 1912 is 1.932 calories per square centimeter per minute.

Subject to the possibility that there may exist ultra-violet rays of appreciable intensity beyond the wave-length $0.29\ \mu$, which are cut off by the absorption of ozone from reaching the earth's surface, we believe that this value represents the intensity of the radiation of the sun as it would be found in space at the earth's mean solar distance for the epoch 1905 to 1912.

In the year 1903 we found indications that the radiation of the sun is not constant from day to day.[2] It has been a main object of the work to ascertain if these apparent variations of the sun are really solar, or are due to some accidental or atmospheric influences not fully eliminated. As early as the year 1910 it had been shown that practically equal solar-constant values were obtained on good days at sea-level, at 1730 and at 4420 meters elevation, and it had been shown that the apparent fluctuations of the solar radiation

[1] See "Smithsonian Pyrheliometry Revised," *Smithsonian Miscellaneous Collections*, **60**, No. 18, 1913.

[2] See *Astrophysical Journal*, **19**, 305, 1903.

found on Mount Wilson from day to day marched by regular steps from high to low values and return, not fluctuating wildly as they would have done had they been due to experimental error. Accordingly it seemed from the first consideration (namely, that altitude did not appear to affect the results) that the atmosphere was not the cause of the fluctuation; and from the second consideration (namely, that the values marched step by step from high to low or vice versa) that it was not an accidental fluctuation. Hence, the most probable conclusion was either that the radiation of the sun is actually variable, or that some meteoric or other matter, by interposition between the earth and the sun, alters the quantity of the radiation received at the earth from day to day. The fluctuations appeared to be of irregular magnitude and period, often ranging through 5 per cent or more, in an interval of 7 or 10 days.

However probable the result just stated might appear, it could not be fully verified without carrying out the observation simultaneously at two stations widely separated on the earth's surface, so that no local atmospheric influences could be supposed to affect both stations at once. This extension of the work was made possible by the Algerian expeditions of 1911 and 1912. Solar-constant determinations were made nearly simultaneously at Mount Wilson, California, and Bassour, Algeria, separated by about one-third of the circumference of the earth. A difference of time of about 8 hours generally occurred between the observations, but inasmuch as the apparent fluctuations of the sun seldom reach 1 per cent in a day, this difference of 8 hours seems not much prejudicial to the comparison.

We were somewhat unlucky in our expeditions. In 1911 a box containing the bolometer and other necessary parts was delayed one month in reaching Algeria, so that a long period of good weather in August was lost. Also the months of September, October, and November 1911 proved less favorable than usual at Mount Wilson and less favorable than had been hoped at Bassour. Thus the number of days in 1911 in which good observations were secured in both places was rather small. In the year 1912, although the sky was generally cloudless, the eruption of the volcano of Mount Katmai in Alaska of June 6 and 7 so filled the sky with haze,

both at Mount Wilson and at Bassour, that a great many days of July and August were rendered unsuitable for comparison between the two stations. Thus it occurred that of 75 days in which observations were secured at both stations in the years 1911 and 1912, only 48 were found good enough for satisfactory comparisons of the solar constant values obtained.

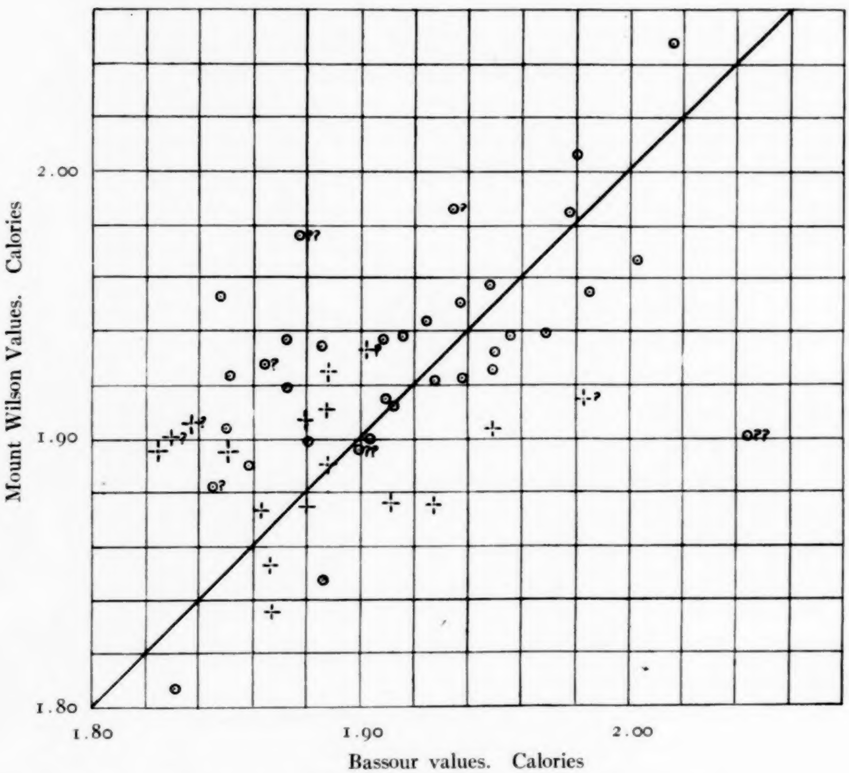


FIG. 1.—Comparison of Bassour and Mount Wilson values of the solar constant

For the purpose in view, namely, to show whether the apparent fluctuation of solar radiation is due to something outside the earth, it is immaterial whether the days of observation are consecutive or not. It is only required to know whether if high values are found at Bassour, high values will occur on the same day at Mount Wilson, and if low values are found at Bassour, low values will be found on

Mount Wilson. It matters not whether the days in question be found in one year or another, provided that they be numerous enough to exclude the probability that an agreement, if obtained, is owing wholly to chance.

The accompanying illustration gives the results of all the days found suitable for comparison between Bassour and Mount Wilson. Ordinates are solar constant values as obtained at Mount Wilson, abscissae are solar constant values as obtained at Bassour. Circles represent the results of days of the year 1912, and crosses represent the results of days of the year 1911. If the solar radiation had varied, and all determinations of it had been free from error, the points must all have lain upon the straight line inclined at 45 degrees to the axis. As it is impossible that results shall be entirely free from error, we must expect that the points representing individual days will be well represented by the 45-degree line if the sun is variable, but will fall uniformly distributed about one point on that line if the sun's radiation is constant. There is no difficulty in deciding that the line and not some single point of the line best represents the results here given.

The variation of the sun shown between the extreme observations amounts to 11 per cent and many observations unite in showing a variation of 7 per cent. The average deviation of the separate determinations at Bassour from those of the same days at Mount Wilson is 1.6 per cent.

Hence the average deviation of a single day of solar-constant measurement at one station will be $\left(\frac{1.6}{\sqrt{2}}=\right)$ 1.1 per cent, and the probable error of a single solar-constant measurement at one station will be 0.9 per cent. Had the condition of the sky in 1912 been free from the haze which prevailed owing to the volcanic eruption of Mount Katmai, we believe the probable error of the separate determinations of 1912 would have scarcely reached 0.5 per cent.

It will be seen that the measurements of 1912 are on the average above those of 1911, at both stations. The difference 1912–1911 is 0.04 calories per square centimeter per minute. This in itself may be regarded as an indication of the variation of the sun depending

upon nearly 20 days of observation in 1911 and about 30 days of observation of 1912.

In further study of the variation of the sun we have compared the mean solar-constant values obtained on Mount Wilson for the different months of the years 1905 to 1912 with the monthly values of the sun-spot numbers as published by Wolfer. We find a fluctuation of solar radiation in the sense that when the sun-spot numbers are high the solar radiation is high and vice versa.

It is also indicated that when the solar radiation is increased the intensity of the violet and ultra-violet rays of the solar spectrum (as it would be found outside the atmosphere) is increased with respect to the intensity of the red and infra-red.

Again it seems to be indicated that when the solar radiation is high the contrast between the brightness of the center and edge of the solar disk is greater than normal.

These and other results of this long investigation are published with details in Vol. 3 of the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, now in press and expected to appear about July 1, 1913. The most important conclusions are as follows:

1. The mean value of the solar constant of radiation for the epoch 1905–1912 is 1.932 calories per square centimeter per minute.

2. An increase of 0.07 calories per square centimeter per minute in the "solar constant" accompanies an increase of 100 sun-spot numbers.

3. An irregular variation frequently ranging over 0.07 calories per square centimeter per minute within an interval of 10 days is established by numerous nearly simultaneous measurements at Mount Wilson, California, and Bassour, Algeria.

4. Indications of two wholly independent kinds incline us to think that these variations of solar radiation are caused within the sun, and not by interposing meteoric or other matter.

Smithsonian Astrophysical Observatory
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THE USE OF THE PHOTO-ELECTRIC CELL IN STELLAR PHOTOMETRY

PRELIMINARY NOTE

By W. F. SCHULZ

The great sensitiveness of the photo-electric cell has been shown experimentally by Elster and Geitel,[1] by Nichols and Merritt,[2] and by J. G. Kemp.[3] From the results of these investigations it seemed that such a cell might be used to measure the light from fixed stars, and its variation. The following is an account of some successful preliminary experiments in an attempt to make such measurements.

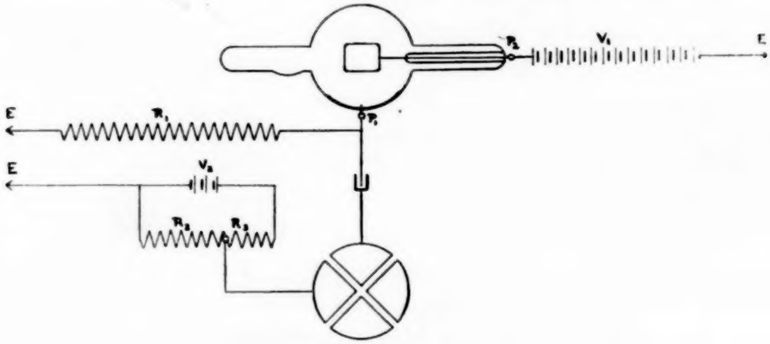

Diagram of apparatus

Several cells of the form shown in the accompanying diagram were prepared in the following way. The anode was a platinum wire about 0.5 mm in diameter, bent into a rectangular loop about $1 \times 1\frac{1}{2}$ cm on the side, the terminal passing through a glass sleeve 3 or 4 cm long. On the wall of the tube facing the plane of this loop was a layer of potassium which formed the cathode. In order to have good contact at the cathode a layer of silver was deposited on and around the platinum terminal on the inside of the bulb. The

[1] *Physikalische Zeitschrift*, **13**, 468, 1912.

[2] *Physical Review*, **34**, 476, 1912.

[3] *Ibid.* (2), **1**, 274, 1913.

bulb proper was about 5 cm in diameter. Potassium was distilled from a similar bulb into a second one, then poured into a pocket in the tube just outside of the bulb of the photo-electric cell and finally distilled upon the silver surface surrounding the cathode terminal.

A little hydrogen gas was then introduced by heating a strip of palladium contained in a side tube. A potential-difference of 560 volts D.C. was applied to the electrodes P_1 and P_2, P_1 being negative, with a lamp resistance of 3000 ohms in series with the cell. When the circuit was closed for a few seconds the bright metallic colors of the hydrogen compound appeared at once on the potassium. There was a uniform soft glow over the entire surface of the metal, the rest of the bulb being non-luminous. It was found necessary to use a rather high potential-difference with a resistance. When a potential-difference of 300 volts with little or no resistance was applied, the discharge took the form of an arc rather than that of a glow, and the current rose rapidly, in one case even melting the electrode.

The circuit was broken when the surface of the potassium had assumed a brilliant violet-blue color and the hydrogen was carefully pumped out and was replaced by a small quantity of helium. All traces of oxygen were removed from the helium by passing it through a tube in which potassium was evaporated, before introducing it into the cell. The photo-electric cell was next connected in series with a sensitive galvanometer and the lamp resistance, and a potential-difference of 300 volts was applied. The light from a small gas flame was allowed to fall on the metal of the cell, and the pressure in the latter was varied by small steps until the galvanometer deflection was a maximum. The tube was then sealed off and proved to be constant for a period of several months.

For measuring very small intensities of light two different methods were used. In the colder winter months, especially in the open observatory, the temperature of the cell was so low that the natural leak through the dark cell was negligible, and the photoelectric current was measured directly by the rate of deflection of a quadrant electrometer. Toward the spring, however, when the temperature rose, the natural leak through the cell increased rapidly

with the temperature, and it was found necessary to compensate this current by means of an independent circuit as shown in the diagram. The anode of the cell was connected to a storage battery of 160 cells, the negative terminal of which was earthed. The cathode was earthed through a high resistance R_1 and connected through a discharge key to one pair of quadrants of a Dolezalek electrometer. In the compensating circuit a battery of 3 cells sent current through a variable resistance R_2R_3 of 20,000 ohms, and the negative terminal was earthed. The other pair of quadrants was connected to R_2R_3 by means of a traveling plug. By this arrangement the "dark current" could be completely neutralized. V_1 was varied from 150 to 320 volts. This was not quite the upper limit at which the cell could be used, but 350 volts was too large, and the photo-electric current reached a value beyond that of saturation. R_1 was a very high resistance of xylol with just a trace of pure alcohol. The sensitiveness of the electrometer was such that a potential difference of 20 volts on the needle and 1.4 volts between the quadrants produced a deflection of 120 mm at a scale-distance of 2 meters. The deflections were very steady. The cell was tested by the light from a small incandescent lamp, which was cut down by passing it through two large crossed Nicol prisms. The cell was mounted in a light-tight box, carefully blackened inside, and closed by means of a shutter. A long closed tube was screwed into the opening of the box, and the lamp placed in this at 1.5 meter distance from the cell. The Nicols were inserted between lamp and cell, with a device for measuring and varying the angle between them. The candle-power of the lamp measured on a two-meter photometer with Lummer-Brodhun screen was approximately 0.003 at 6 volts. The deflections of the electrometer were easily read even when the planes of polarization made an angle of 85° with each other. The intensity of the light which passed through an opening of 1 sq. cm area at the cell was therefore $0.003 \times \cos^2 85/1.5^2 = 0.000010$ candle meters.

It has been shown by Ångström that the energy flowing from an amyl acetate lamp is approximately 10^{-8} gram calories per square cm at a distance of 1 meter. Assuming the Hefner unit and the

candle-power to be equal and the distribution of energy to be the same in both lamps, we find that the quantity of energy incident on the cell is approximately 0.000010×10^{-8} gram calories or 4.19×10^{-6} ergs. This produces a deflection of the electrometer which is easily read. So far the light from two stars has been measured by means of this cell; in December 1912, that of *Capella* and in April 1913, that of *Arcturus*. The cell in its light-tight box was mounted on the 12-inch equatorial at the observatory of the University of Illinois, and placed in such a position beyond the focal plane of the objective that the circle of illumination on the sensitive surface of the cell had an area of about 1 sq. cm.

On the cold December nights the natural leak through the cell was almost zero, and the photo-electric current was measured by the rate of the electrometer deflection. With 40 volts on the needle and 160 volts on the cell, the rate of deflection at a scale-distance of 2 meters was 20 mm in 30 seconds. With 200 volts on the cell, the rate was 18 mm in 20 seconds. These deflections were repeated without difficulty.

In April 1913, another set of readings was taken with the light from *Arcturus*. This time the dark current had to be compensated. With 60 volts on the needle and 250 on the cell, the deflection due to the photo-electric current alone was 22 mm. This was reduced to zero each time by varying the resistance R_3. With 60 volts on the needle and 300 on the cell, the deflection when the cell was exposed to the light of *Arcturus* was 48 mm; with 60 volts on the needle and 325 on the cell the deflection was 190 mm; and with 80 volts on the needle and 325 on the cell the deflection was 248 mm. The sensitiveness of both cell and electrometer can be increased.

These measurements seem to show that it is possible to use the photo-electric cell for astrophysical investigations. The present research is being continued along various lines. It is planned to compare the sensitiveness of the photo-electric cell with that of the selenium cell, and to study the influence of temperature upon the "dark current," the effect of the wave-length of the incident light upon the lower limit of sensitiveness, and the use of various alkali metals for the sensitive layer.

These measurements were made at the suggestion of my friend Dr. Jakob Kunz, to whom I am deeply indebted for the benefit of his invaluable experience in making the cells and for assistance in conducting the experiments.

UNIVERSITY OF ILLINOIS
May 19, 1913

Minor Contributions and Notes

ARE DIFFRACTION PHENOMENA POSSIBLE AT SOLAR ECLIPSES?

At the beginning and at the end of solar eclipses, moving light and dark bands become visible on the ground, the so-called "flying shadows." Among the possible explanations of their appearance, an attempt to represent them as diffraction phenomena is worth considering, since not only the magnitudes but also the distances of the objects are considered in such an explanation.

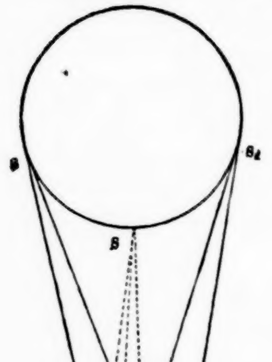

FIG. 1

To make a test of this form of explanation, I have used the diagram in Fig. 1. S represents a luminous point of the sun's surface, L the center of the moon, E one of the observed points on the earth's surface located on the line SL. We take a system of three rectangular axes X, Y, Z, of which the XY plane and X axis pass through L perpendicular to the line SE. The moon will then appear as a dark flat screen in the XY plane. Instead of this round screen, we may substitute the square screen $V_1V_2U_1U_2$ circumscribed about it, with sides parallel to the axes. On account of the symmetry, the phenomenon need only be studied in the XZ plane along the line EP which is perpendicular to SE. For a particular point P of this line the origin of coordinates, O will be taken as the intersection of PS with the X axis.

By Fresnel's theory of diffraction the intensity I in P is given by

$$I = A_1^2(C^2+S^2)$$

where $A_1 = \dfrac{A\cos\phi}{\lambda\rho_0\rho_1}$, A = amplitude in S, $\phi = ZOS$, $\rho_0 = OP$, $\rho_1 = OS$, λ = wave-length. Fresnel's integrals C and S are to be taken over the entire free part of the XY plane

$$C = f\left\{\left[\int_{-\infty,\,v_2}^{v_1,\,+\infty}\cos\frac{\pi}{2}v^2dv\int_{-\infty}^{+\infty}\cos\frac{\pi}{2}u^2du - \int_{-\infty,\,v_2}^{v_1,\,+\infty}\sin\frac{\pi}{2}v^2dv\int_{-\infty}^{+\infty}\sin\frac{\pi}{2}u^2du\right]\right.$$

$$\left.+\left[\int_{v_1}^{v_2}\cos\frac{\pi}{2}v^2dv\int_{-\infty,\,+u}^{-u,\,+\infty}\cos\frac{\pi}{2}u^2du - \int_{v_1}^{v_2}\sin\frac{\pi}{2}v^2dv\int_{-\infty,\,+u}^{-u,\,+\infty}\sin\frac{\pi}{2}u^2du\right]\right\},$$

$$S = f\left\{\left[\int_{-\infty,\,v_2}^{v_1,\,+\infty}\sin\frac{\pi}{2}v^2dv\int_{-\infty}^{+\infty}\cos\frac{\pi}{2}u^2du + \int_{-\infty,\,v_2}^{v_1,\,+\infty}\cos\frac{\pi}{2}v^2dv\int_{-\infty}^{+\infty}\sin\frac{\pi}{2}u^2du\right]\right.$$

$$\left.+\left[\int_{v_1}^{v_2}\sin\frac{\pi}{2}v^2dv\int_{-\infty,\,+u}^{-u,\,+\infty}\cos\frac{\pi}{2}u^2du + \int_{v_1}^{v_2}\cos\frac{\pi}{2}v^2dv\int_{-\infty,\,+u}^{-u,\,+\infty}\sin\frac{\pi}{2}u^2du\right]\right\},$$

where $f = \dfrac{\lambda\rho_0\rho_1}{2(\rho_0+\rho_1)\cos\phi}$, $v = x\cos\phi\sqrt{\dfrac{2(\rho_0+\rho_1)}{\lambda\rho\cdot\rho_1}}$, $u = y\sqrt{\dfrac{2(\rho\cdot+\rho_1)}{\lambda\rho\cdot\rho_1}}$,

$$x_1 = OV_1,\; x_2 = OV_2,\; y = LU_1 = LU_2.$$

Now the wave-length of light is $\lambda = 5.10^{-10}$ km, the radius of the moon $y = 1741$ km. When $\cos\phi = 1$, the distance from sun to moon $\rho_1 = a = 148{,}642{,}930$ km, the distance from the moon to the earth, $\rho_0 = b = 350{,}703$ km; thus we have $u = 186{,}152$. With arguments as big as the preceding ones we have

$$\int_{-u}^{+u} = \int_{-\infty}^{+\infty} = 1 \text{ and therefore } \int_{-\infty,\,+u}^{-u,\,+\infty} = \int_{-\infty}^{+\infty} - \int_{-u}^{+u} = 0.$$

Since we need investigate the diffraction only for points P for which the origin of co-ordinates O is close to the limit of the moon, we have

$$\int_{v_1}^{+\infty} = \int_{0}^{+\infty} - \int_{0}^{v_1} = 0.$$

The intensity at P is then

$$I = \frac{I_0}{2}\left[\left(\int_{-\infty}^{v_1} \cos\frac{\pi}{2}v^2 dv\right)^2 + \left(\int_{-\infty}^{v_1} \sin\frac{\pi}{2}v^2 dv\right)^2\right]$$

where $I_0 = \frac{A^2}{(a+b)^2}$ represents the intensity at P without the screen. The simplifications which present themselves here mean physically that we are concerned only with diffraction phenomena at the limb of the moon. The geometric interpretation of the expression for I shows that $I = \frac{I_0}{2}\left(-\infty, v_1\right)^2$, i.e., proportional to the square of distance between the asymptotic point $-\infty$ and the point v_1, of Cornu's double spiral. If we take the point P as E and therefore O as L, then $v_1 = -186.152$

$$(-\infty, -186{,}152)^2 = 0 \text{ and } I = 0.$$

This means that there is darkness in the central band, although with a screen of smaller size, there would always be light in the axis of the geometrical shadow. If the point P is shifted from E to F, i.e., O from L to V_1, the intensity will increase continually and at F, on the edge of the geometric shadow of the point P we have

$$I = \frac{I_0}{2}(-\infty, 0)^2 = \frac{I_0}{4}.$$

Shifting the point P outside of the geometrical shadow, maxima and minima will occur, maxima for $v_1 = \sqrt{3/2 + 4h}$, and minima for $v_1 = \sqrt{7/2 + 4h}$, $(h = 0, 1, 2, \ldots)$.

If, instead of $OV_1 = x_1$ the length $PF = d$ is introduced from the proportion $\frac{x_1}{d} = \frac{a}{a+b}$ we have

$$v_1 = x_1 \sqrt{\frac{2(a+b)}{\lambda ab}} = d \sqrt{\frac{2a}{\lambda b(a+b)}} \, .$$

Thus maxima and minima will occur at the following distances from the edge of the geometrical shadow of the point S.

Maxima : $d = \sqrt{\frac{(3+8h)\lambda b(a+b)}{4a}}$ Minima : $d = \sqrt{\frac{(7+8h)\lambda b(a+b)}{4a}}$

With the numerical values which we have adopted, the first maximum will be at a distance $FP = d = 11.5$ m; the next minimum at $FP = d = 17.5$ m. The intensity of light at the first maximum is $I = 1.34$; of the next minimum $I = 0.78$, when $I_0 = 1$ is the free intensity from the point S of the sun. So far only one point of the sun has been considered. With a finite extension of the source of light, the individual diffraction patterns will be shifted with respect to each other; the resulting effect will be the superposition of the individual patterns in such a way that for a shift greater than the diffraction, all maxima and minima will have faded out. In our case, the shifts for points S_1 and S_2 in comparison with S is equal to $FG = FH = \frac{R \cdot b}{a} = 1461$ km where $R = 695740$ km.

Thus diffraction phenomena are excluded in the case of eclipses of the sun on account of the relative magnitudes of the quantities involved. In the umbra from E to F the intensity increases continuously and in the penumbra from F to H proportionally to the apparent part of the surface of the sun which is visible. Outside of the penumbra beyond H, where the bands originated by S_2 are not reached by the bands due to the preceding points, diffraction bands would be possible. They would be colored and in position parallel to the limits of the penumbra, violet on the inside, red on the outside, at distances as given above, but the full light of the sun would make them completely invisible.

Since beyond F the first maximum for $\lambda=3.8\times10^{-10}$ km will appear at $d=10.2$ m and for $\lambda=7.6\times10^{-10}$ km at $d=14.2$ m, the bands will be colored. By the superposition of the diffraction patterns of neighboring points of the sun, white light will result first at $d=14.2$ m, while from $d=10.2$ to $d=14.2$, the light starting with blue will pass through mixed colors finally into white. The inner edge of the umbra should thus show a bluish lining.

The relative size of the quantities here encountered will also appear in other systems, e.g., eclipses of the sun in Jupiter's system. Here, too, the shadows of the satellites will project themselves without diffraction on the surface of Jupiter.

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THE MELTING-POINT OF MOLYBDENUM

In connection with the work discussed in an earlier article[1] a direct determination has been made of the melting-point of molybdenum by the V-method,[2] using pure ductile material kindly furnished by the General Electric Co. The strips were electrically heated *in vacuo*, and were cut slightly tapering from each end to the middle as viewed from the side, in order to predetermine the region of melting. Observations in the V and on the side with calibrated optical pyrometers gave the following (uncorrected) values for the true and black-body melting temperatures:

	True Temperature	Black-Body Temperature ($\lambda=0.658\mu$)	Weights
1	2495° C.	2197° C.	2
2	2494	2215	6
3	2485	. . . .	1
4	2512	2221	10
5	2506	2212	5
6	2496	2197	2
7	2485	2197	1
8	2512	2199	For true 6 For black-body 2
Weighted means.	2504° C. ± 8°	2212° C.	

[1] *Astrophysical Journal*, **37**, 380, 1913.

[2] *Ibid.*, **33**, 91, 1911.

The correction for the absorption of the glass window is 30°.6 for 2500° and 24°.5 for 2212°, which makes for the true temperature of the melting-point of molybdenum **2535° C.**, and 2237° C. for the black-body melting-point, for $\lambda = 0.658\ \mu$.

These temperatures are on the basis of $c_2 = 14{,}500$ and the palladium melting-point at 1549° C. The weights were determined by the working conditions. In all cases, though sublimation is rapid in a vacuum, the ends of the filaments were evidently melted, and up to the time of melting no particular non-uniformity of temperature developed in the strips.

For comparison, this result is combined in the following table with others quoted from Pirani and Meyer:[1]

MELTING-POINTS OF MOLYBDENUM. DEGREES C.

Waidner and Burgess	v. Wartenberg	Ruff and Goecke	v. Pirani and Meyer	M. and F.
2500(?)	2521	2123	2393	2535

The figure quoted from Waidner and Burgess[2] is apparently only an estimate, that of v. Wartenberg is based on only two rather discordant (2440°, 2530° C.) observations in a vacuum tungsten tube furnace, and is merely given as ">2500," which, moreover, must be raised 20° for comparison with ours, because based on 1745° C. for the platinum melting-point. Pirani and Meyer's value is computed from the mean of two concordant observations of the black-body melting-point, assuming a constant value (0.51) for A_λ. If this value for the melting-point is to be compared with ours, it must also be increased about 15°, as they also used 1745° C. as the melting-point of platinum. Making this correction, and computing S from their published value of T, it follows that their observation of the black-body melting temperature must have agreed very closely with ours. We have not had access to the original paper of Ruff and Goecke. The greatest outstanding uncertainty, which we intend to study further, is undoubtedly as

[1] *Verh. der deutschen phys. Ges.*, **14**, No. 8, 1912.

[2] Burgess, *Measurement of High Temperatures* (1912), p. 492.

to whether the melting-point *in vacuo* is, because of rapid sublimation, lower than that in some neutral gas at atmospheric pressure. Aside from this possibility, our experience with molybdenum indicates that it is well adapted to furnish a standard high-temperature reference point.

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February 1913

NOTE ON THE TRANSMISSION OF THE ATMOSPHERE FOR EARTH RADIATION

In connection with the problem concerning the effective radiation from the earth's surface, it is of interest to regard the question of the transmission of the atmosphere for the earth radiation.

Very[1] has claimed a value of 40 per cent for the transmissive power of the atmosphere.

Abbot and Fowle,[2] on the other hand, are of the opinion that the atmosphere transmits only about one-tenth of the radiation from the liquid and solid surface of the earth, a conclusion that they found upon the measurement of Rubens and Aschkinass on the absorption of the water-vapor.

Without going into details regarding these considerations, we may here look for the conclusion that can be drawn from the direct measurements of the radiation to space.

The earth radiates for the wave-lengths with which we here are dealing almost as a black body, and should, if no atmospheric radiation existed, radiate to space $0.490 \frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$, the temperature assumed to be 283° absolute.

The observed mean radiation (the effective radiation) is only about 0.15 (probably somewhat less, taking in regard the saturated water-vapor over the water surfaces), and if the atmosphere could be regarded as having uniform density and a temperature equal

[1] *Astrophysical Journal*, **34**, 374, 1911.

[2] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 1908.

to that of the earth's surface, we should conclude that the transmissive power of the atmosphere is about 30 per cent.

But now the temperature of the atmosphere is falling off with the height above sea-level and a part of the observed nocturnal radiation is not transmitted entirely through the atmosphere, but merely to its colder layers. The effective radiating layer of the atmosphere may be regarded as having a certain temperature T, which is less than the temperature of the surface of the earth. It is difficult to give a definite value for this temperature T. From the observations of Pernter and Trabert[1] on the atmospheric radiation at different heights above sea-level, we may, however, conclude that a very small part of the radiation that reaches the earth from the atmosphere has its origin in layers lying at a greater altitude than about 3000 m above sea-level. At a height of 3000 m (top of Sonnblick) they find the atmospheric radiation to be only about $0.12 \frac{\text{gr. cal.}}{\text{cm.}^2 \text{ min.}}$ and since the atmosphere absorbs strongly the wave-lengths which it itself radiates, it is probable that only a very weak part of this radiation reaches the earth's surface. The temperature-fall being about $0\overset{\circ}{.}07$ for 100 m, the temperature at 3000 m will be about 21° below that at sea-level and we may assign a mean value for the temperature of the effective radiating layer which lies about 10° below the temperature of the earth's surface. This assumption, which cannot be very far from the real conditions, leads us to consider about 0.06 gr. cal. of the nocturnal radiation as intercepted by colder layers of the lower atmosphere. Of the remaining 0.09 gr. cal. which probably are transmitted through the lower absorbing layers, a considerable part will be absorbed by the ozone in the higher and colder strata of the atmosphere. As has been shown by K. Ångström,[2] the ozone has a strong absorption band between 9 and 10 μ, which is in the region where the water-vapor is weakly absorbing. In the winter time, when the quantity of ozone is relatively great, this absorption will reach about 50 per cent in the region of selective absorption; in the summer time the absorption is

[1] *Sitzungsber. d. d. Wiener Akad.*, **47**, 1562, 1888.

[2] *Arkiv för matematik, astronomi och fysik*, p. 347 (1904); p. 395 (1904) (German language).

almost zero. We may, however, as a mean value put the absorbing power of the upper strata of the atmosphere equal to about 20 per cent, and we shall in this way come to the conclusion that about 0.07 gr. cal. or only about 14 per cent of the radiation from the earth's surface will escape the absorbing atmosphere and go out to space. This is under the assumption that the sky is clear. This value for the transmission, which is not far from that derived by Abbot and Fowle from the measurements on the absorptive power of the water-vapor, is naturally subjected to some variation and must be regarded as only approximate.

Our considerations support us, however, in the belief that the transmission for clear sky seldom is greater than 25 per cent and seldom is less than about 5 per cent.

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December 1912

ON THE LONG-PERIOD VARIABLE STARS

In vol. **67** of the *Monthly Notices* (1907) Professor H. H. Turner has discussed the light-curves of some long-period variable stars. He starts from the generally acknowledged theory, which explains the light-fluctuations of these stars as being caused by periodic variations in the number and extension of sun-spots and faculae in their photospheres. Turner supposes that the "Spoerer law" of the distribution of sun-spots is valid for all these heavenly bodies. Then he shows how the orientation in space of the axis of rotation of a star will influence the type of the light-curve of the star. The extreme cases occur when the star is seen directly from its pole and when it is seen equatorially. Turner at last gives, as a distinguishing feature, the formula

$$a=\frac{2(M-m)-P}{P},$$

where M signifies the time of maximum, m, the time of the preceding minimum, and P, the period. Then the value of a indicates the orientation of the axis of rotation with respect to the line of sight. a may vary between the limits $+0.25$ and -0.70. A large value of a indicates that the star is seen essentially from the

pole, while a small value of α will be found if the star is seen nearly equatorially. Turner applied this formula to a considerable number of the known long-period variable stars, and he arrives at the interesting conclusion that the axes of these stars are roughly oriented parallel to the plane of the Milky Way. I have made some further investigations concerning this matter in order, if possible, to discover any new relation between the given quantities. My thought has been, briefly expressed, that the orientation of the axis of rotation of the star must influence its light-curve in another way than by displacing the time of the maximum with respect to that of the minimum. The difference between the intensities of the light at maximum and at minimum must be much greater in case of an equatorial view than in case of a polar view when, as we know, everything will be presented along the edge of the disk of the star and, as Turner indicates, is seen very greatly foreshortened and when the brightness at maximum (at any rate, when caused by faculae) is essentially decreased by the absorption in the atmosphere of the star. He has, as far as I can see, not made use of this fact in the final discussion. He gives the data for such an investigation in the two lists of stars having large and small values of α. The only thing that remains to be done is to write beside the given values of α the values of $B-b$, where B signifies the maximum and b the minimum brightness of the star in question expressed in star magnitudes. This I have done and I find the values given in the following tables. The values of B and b are taken from the list of elements of variable stars given in the *Annuaire du Bureau des longitudes* for the year 1909.

α	Star-Names	$B-b$	*R.A.*
+0.25	*X Ophiuchi*	2.1	18^h34^m
0.22	*V Cephei*	0.7	23 52
0.19	*RS Librae*	6.0	15 18
0.18	*W Scorpii*	4.1	16 6
0.17	*V Ophiuchi*	2.9	16 21
0.16	*S Carinae*	3.1	10 6
0.13	*S Arietis*	6.6	1 59
0.12	*T Cassiopeiae*	4.0	0 18
0.11	*T Capricorni*	4.2	21 17
0.10	*S Cephei*	3.6	21 36
0.08	*T Cephei*	3.7	21 8
0.08	*V Tauri*	4.7	4 46

α	Star-Names	$B-b$	R.A.
+0.07	*X Capricorni*	6.2	$21^h\ 3^m$
0.07	*W Cygni*	[illegible].8	21 32
0.07	*U Boötis*	3.3	14 50
0.05	*V Cygni*	5.4	20 38
0.05	*R Camelopardalis*	5.6	14 25
0.03	*R Aurigae*	5.9	5 9
0.03	*W Cassiopeiae*	3.6	0 49
0.02	*R Sagittarii*	2.0	19 11
−0.19	*T Arietis*	1.3	2 43
0.19	*S Piscium*	6.0	1 12
0.20	*V Boötis*	3.0	14 26
0.20	*U Orionis*	6.0	5 50
0.22	*R Aquilae*	4.6	19 2
0.22	*S Hydrae*	4.1	8 48
0.22	*Y Virginis*	5.3	12 29
0.23	*R Canis minoris*	2.3	7 3
0.25	*o Ceti (Mira)*	5.5	2 14
0.26	*R Centauri*	6.1	14 9
0.27	*U Cassiopeiae*	7.0	0 41
0.27	*R Cygni*	7.1	19 34
0.27	*V Lyrae*	6.4	19 5
0.27	*R Ursae majoris*	5.8	10 38
0.31	*R Cancri*	5.0	8 11
0.34	*R Comae*	6.3	12 0
0.34	*S Coronae*	5.3	15 17
0.35	*R Geminorum*	6.3	7 1
0.42	*R Andromedae*	8.7	0 18
0.63	*S Tauri*	4.0	4 24 (value of α doubtful)

If we divide these stars in groups and take the means of $B-b$ we shall have the following table:

α	$B-b$	Number of Stars in the Group
+0.20 to +0.25	1.40	2
+0.15 +0.20	4.03	4
+0.10 +0.15	4.60	4
+0.05 +0.10	3.74	5
+0.00 +0.05	4.50	5
−0.19 −0.24	4.06	8
−0.24 −0.29	6.31	6
<−0.29	5.93	6

And really, the small values of α correspond with large values of $B-b$ and inversely. This result seems to me very interesting. It shows the general supposition of the cause of the light-variations of these stars to be correct. Professor Turner's beautiful theory

has given us the means of verifying this hypothesis in another way than that usually followed by the modern astrophysicist who if possible makes his investigations by means of spectrum analysis. As we know, the spectroscope has revealed the great accordance of the spectra of the sun-spots with those of the stars of the third type, to which the majority of the long-period variable stars belong. If we acknowledge that this accordance demonstrates that the variation of the brightness of these stars is caused by increments and decrements of the "solar activity" on these heavenly bodies, we must hereafter acknowledge Turner's results to be nearly correct, as the criterion found above is fulfilled. This fact is of the greatest cosmogonical interest. The next step forward in this matter may be to inquire if the axes of rotation of these stars are oriented haphazard in planes roughly parallel to the galaxy or if they favor any direction in these planes. I do not think, however, that the material at present at our disposal, consisting of records of stars of this type lying in the plane of the Milky Way, is sufficient for such a delicate inquiry.

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April 1, 1913

Reviews

Conférences sur quelques thèmes choisés de la chimie physique pure et apliquée. Par SVANTE ARRHENIUS. Paris: Librairie Scientifique A. Hermann et Fils, 1912. Pp. 112; figs. 14; tables 7.

In this brochure of 112 pages there are brought together five lectures recently delivered at the Sorbonne by the distinguished Swedish savant. As may be inferred from their titles, the five essays cover a wide range of subjects: (I) "La théorie moléculaire"; (II) "Les suspensions et les phénomènes d'adsorption"; (III) "L'Énergie libre"; (IV) "Les atmosphères des planètes"; (V) "Les conditions physiques sur la planète *Mars*."

The first lecture is essentially a historical sketch which commences with the Greek philosophers Leucippus and Democritus and traces the development of the molecular theory to its present status. The chief interest naturally lies in the more recent advances, and here Arrhenius shows a superficial reading of the subject. If he had only read the résumé in the *Jahrbuch der Radioaktivität* he would have known that Ehrenhaft's critique of Millikan does not hold, for the variation in the observed readings was not due to any variation in the charge itself, which Millikan has since shown to be constant, but to variable readings caused by Brownian movements. Upon this apparent irregularity as an established foundation, Arrhenius works up a possible superstructure of theory. But that superstructure falls as the foundation is removed. As brought out by Arrhenius himself at the close of the chapter, the latest researches of Millikan have corrected this irregularity and have established the constancy of the electric charge. The wonder is that so much consideration was given to this irregularity.

The discussions of the second and third lectures are chiefly chemical in nature and are less within the field of astrophysics than the treatment of the atmospheres of the planets in the fourth lecture and the physical conditions on the planet *Mars* in the fifth. The general discussion in both of these last is based upon the theory of Laplace and stands or falls as that famous hypothesis withstands or goes down before the very grave objections which have arisen in recent years. To those who still

believe in the Laplacian hypothesis, much of this discussion may be acceptable; but to those who do not accept it as law and gospel, but rather regard it as now on trial for even a plausible standing, the conclusions so confidently and complacently built upon it seem no more secure than the foundation.

The treatment of terrestrial atmospheres and early terrestrial conditions is a restatement of the view made familiar long ago by Dana and others, with a few modern qualifications. The author either seems to be unaware of the difficulties confronting that time-honored view which have been pointed out in recent years, and the alternatives which have been proposed to meet some of these difficulties, or else prefers to ignore them and plunge ahead with free and familiar phrasing. He slides easily by the difficulty of the earth's incapacity to hold at any one time sufficient hydrogen, hydrocarbons, and cyanogen to produce, by combination with oxygen, the waters of the globe, which in the vaporous state would amount to at least 200 times the present atmosphere, and the carbon dioxide of the coal and limestone beds, which would amount to perhaps 50 times the earth's present atmosphere. He ignores the difficulty of accounting for the free oxygen which united with the hydrogen, hydrocarbons, and cyanogen to furnish the water and carbon dioxide; as well as the difficulty of starting vegetable life on a globe surrounded by an oxygenless atmosphere. He seems to see no infelicity in retaining carbon dioxide and other gases in large quantities within a globe which has been subjected to long-continued boiling before a solid crust formed upon it.

The gloomy picture of the "dead planet" *Mars*, supposed to have lost most of its atmosphere, is held up before us as the image of our globe in its period of decline. A discussion of the so-called canals comes to the front. In seeking rectilinear lines on the earth to compare with the supposed canals on *Mars*, Arrhenius seizes upon the geotectonic lines. If really carried to its proper geological conclusion, this leads to the hypothesis that the canals are analogous to such terrestrial features as the rift valleys like the Red Sea and Lake Tanganyika, a conception which has already been entertained by others.

The question of whether the faulted valleys are partially filled with water as on the earth and hence in any sense "canals," is pretty effectually answered by Campbell's careful observations on the summit of Mt. Whitney where, under very favorable conditions, he failed to detect a trace of water vapor in the Martian atmosphere.

The yellowish areas on the surface of the planet *Mars* are explained by Arrhenius as due to desert sands analogous to the great Khévir in

Persia. The dark bluish-green color assigned to the canals and Martian lakes he supposes may be due to the action of gaseous emanations along the fissures. He suggests that hydrogen sulphide and other reducing agents escaping from fissures might have transformed the iron salts in the neighborhood of these lines into bluish-green sulphides.

R. T. CHAMBERLIN

Radioactive Substances and Their Radiations. By E. RUTHERFORD. New York: Putnam, 1913. Pp. i–vii+1–700. $4.50.

A new book on radioactivity written by the foremost authority in this field, and after a lapse of eight years since the appearance of the last edition of his former treatise, is almost as notable an event as the discovery of a new radioactive product. But despite the fact that Rutherford has written what is practically a new book, instead of merely revising the old one, and despite the fact that the subject is now just twice as old as it was when the former book was written, the present treatment does not show anywhere any radical change of front in the interpretation of radioactive phenomena. This is because radioactivity, like the other grand divisions of physics, has probably already passed the period of revolutionary changes and is likely to grow henceforth by the process of accretion. In other words, the correct interpretation of the main radioactive phenomena has doubtless already been given.

The book in hand, then, differs from its predecessor chiefly in incorporating the advances of the past eight years without omitting much of what the other book contained. These advances have had to do chiefly with increases in the amount and accuracy of our knowledge of the radiations from active substances, the nature of their absorption by matter, and of their connection with the transformations; with the discovery of methods of counting single α particles; with the discovery and careful study of the phenomena of recoil; and with the extension through these methods of the known products of radioactive change from 20 in 1905 to 32 in 1913.

The chapter headings are: (i) "Radioactive Substances"; (ii) "Ionization of Gases"; (iii) "Methods of Measurement"; (iv) "Alpha Rays"; (v) "Beta Rays"; (vi) "Gamma Rays"; (vii) "Properties of the Radiations"; (viii) "Continuous Production and Decay of Radioactive Matter"; (ix) "Radioactive Gases"; (x) "Active Deposits"; (xi) "Theory of Successive Transformations"; (xii) "Uranium, Ionium and the Origin of Radium"; (xiii) "Radium and Its Emanation"; (xiv) "Active Deposit of Radium"; (xv) "Actinium and Its Products"; (xvi) "Thorium

and Its Products"; (xvii) "Production of Helium and Emission of Heat"; (xviii) "General Results and Relations"; (xix) "Radioactivity of the Earth and Atmosphere." This book will unquestionably be the standard work on radioactivity for the next half-dozen years.

R. A. MILLIKAN

Die Mathematik im Altertum und im Mittelalter. Von H. G. ZEUTHEN. Berlin und Leipzig: B. G. Teubner, 1912. Pp. 95. M. 3.

This monograph is one of six mathematical papers that are to appear under the editorship of Felix Klein of Göttingen, as part of the voluminous German publication known as "Kultur der Gegenwart."

With Zeuthen, Greek mathematics has been a favorite study. Over a quarter of a century ago he published a book on the conic sections in antiquity. In 1893 appeared the first edition of his history of ancient and mediaeval mathematics. Always known as an accurate writer, Zeuthen embodies in the present brief monograph the results of his mature scholarship. The fruits of recent historical research are carefully noted, such, for instance, as the use of symbols for zero in Babylonia at an earlier period than in India, the evolution of our + sign from the florescent *et* of Latin manuscripts, an account of the book on *Metrica* of Heron discovered by Schöne and published in 1903, a description of a book on *Method* due to Archimedes. This book on *Method* was supposed to have been irretrievably lost. The discovery of a copy of it by Heiberg in 1906 in Constantinople marks the most important advance in our knowledge of Greek mathematics in recent years. The manuscript in question is of parchment containing a tenth-century copy of the works of Archimedes. An attempt had been made to wash out the old script; the parchment was then used for liturgical writing. Fortunately the earlier writing shows with more or less clearness on most of the 177 leaves. Among other works of Archimedes, the parchment contains the *Method,* which is of interest to us as disclosing for the first time the steps by which Archimedes worked his way to the discovery of his great theorems with their rigorously scientific proofs. The *Method* affords a glimpse of the processes employed in the workshop of the greatest of ancient mathematicians; it rests upon the use of infinitesimals and of steps akin to those of the integral calculus.

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